Description of the Advanced Mathematics Programs and Curriculum

France

The Grade 11 and 12 scientific track offers robust mathematical knowledge and skills to students aiming for careers in science, technology, engineering, and mathematics (STEM). The mathematics curriculum is meant to develop students' scientific thinking and strengthen their interest in and affinity for scientific research. Together with introducing new mathematical knowledge and content, the curriculum targets developing students' skills and mathematical faculties in these areas:

- Implementing mathematical investigations and employing a variety of problem solving strategies
- Mastering a wide range of reasoning processes
- Interpreting and validating mathematical results
- Communicating mathematics both orally and in writing

Mathematical activities assigned to students both in class and for homework are focused on intra-mathematical or contextually diverse problem solving situations. Students are trained in:

- Searching for information, experimenting, and modeling, all using technology
- Choosing and executing calculation techniques
- Implementing algorithms
- Reasoning, proving, and validating results
- Explaining an answer, communicating a result

The mathematical content is organized in three parts: Analysis, Geometry, Probability and Statistics. About half of class time should be devoted to Analysis, one quarter to Geometry, and the last quarter to Probability and Statistics. The topics included in each content area are listed below.

Content Area	Topics
Analysis	Quadratic functions: solving quadratic equations, sign of a quadratic function Sequences: arithmetic and geometric sequences, induction, finite or infinite limits, bounded sequences Function limits: finite or infinite limits, limits of a sum, product, quotient or composite functions, asymptotes Continuity on an interval, including the Intermediate Value Theorem Differentiation: calculating derivatives, including the derivatives of common functions, derivatives of sums, products, and quotients of functions, and applications of derivatives, including the relationship between the intervals over which a function increases or decreases and the value of its derivative on those intervals and function extrema

Content Area	Topics
Analysis (Continued)	Sine and cosine functions Exponential functions Natural logarithms Integration on an interval, including the relationship between the definite integral and the area under a curve, notation, the antiderivative of a function, linearity, and the additive property of definite integrals
Geometry	Complex numbers, including the algebraic form, conjugate, geometric representation, and polar form of a complex number; the sum, product, and quotient of complex numbers, complex solutions to quadratic equations Euclidean vectors, including the characterization of a line and a plane, scalar product, coordinates, equation of a plane Trigonometry, including trigonometric functions defined on the unit circle, radian, the sine and cosine of supplementary and complementary angles
Probability and Statistics	Descriptive statistics, including variance, standard deviation Conditional probability, independence Probability density functions, including discrete and continuous random variables, probability distributions (normal, Bernoulli, binomial, uniform, exponential), variance, standard deviation Confidence intervals Sampling, confidence interval for a proportion

Starting in Grade 10, scientific track students continue to develop and implement algorithms. Students are trained to:

- Describe algorithms in natural or symbolic language
- Devise basic algorithms using spreadsheets, calculators or specific software programs
- Interpret complex algorithms

Algorithms fit naturally in all mathematical fields. Algorithmic problem solving in each content area is situated in contexts related to academic subject areas and contexts from real life. Students learn how to implement elementary instructions, loops, and conditional instructions as well as to implement validation and control steps in their programs.

Students learn how to use formal mathematical notation (e.g., for functions, derivatives, and integrals) as well as notation for number sets and intervals.

Students learn elements of formal logic, such as the logical operators for "and" and "or"; the concepts of the contrapositive, the converse and the negative of a conditional statement; logical equivalence; types of arguments, such as the counterexample, the logical disjunction, and the contrapositive; and proof by contradiction.

Italy

Students assessed in TIMSS Advanced 2015 have been taught according to the 2010 National Guidelines for upper-secondary schools (*Licei, Istituti Tecnici, Istituti Professionali*). Only students of *Liceo Scientifico* (high schools specializing in science education) and of *Istituto Tecnico–Settore Tecnologico* (technical high school–technology sector) participated in the TIMSS Advanced 2015 advanced mathematics assessment. In fact, these are the only tracks with elements of advanced mathematics in their curricula, with an appropriate lesson time (4 hours per week, for 33 weeks in a school year).

The mathematics curriculum for upper-secondary school includes four main areas:

- Arithmetic and Algebra
- Geometry
- Relations and Functions
- Statistics and Probability

Liceo Scientifico focus on the study of the link between the scientific and the humanistic traditions. They promote the acquisition of knowledge and methods of mathematics, physics, and the natural sciences. At the conclusion of the program of study, students should be able to:

- Understand the formal language of mathematics, know how to use typical mathematical procedures, and know the basic content of the theories underling the mathematical description of reality
- Use data processing tools critically and in-depth; understand the methodological value of information technology in formalizing and modelling complex processes; and identify procedures that lead to conclusions and judgments about real-world systems modelled by data
- Understand the fundamental structures of mathematical argumentation and demonstrate mathematical processes through the mastery of the language of formal logic; use mathematical argumentation and formal logic to identify and to solve problems of various kinds
- Know how to use computational and representation tools for modelling and solving problems

At the end of the course, students of *Liceo Scientifico* must know the basic concepts and methods of mathematics and apply them to describe and predict phenomena in the physical world. They can situate mathematical theories in historical context and understand their conceptual meaning. The *Liceo Scientifico* five-year curriculum is divided into three parts by grade. The topics taught at each grade are listed below.

Grade	Topics
Grades 9 and 10	Arithmetic: integer, rational and real numbers; algebra, polynomials, algebraic equations of first and second-degree, inequalities, simultaneous equations Functions: Linear functions $f(x) = ax + b$, quadratic functions $f(x) = ax^2 + bx + c$, $f(x) = \|x\|$, and $f(x) = a/x$ Euclidean geometry and Cartesian plane geometry: geometric transformations in the plane, circles, circumference, and π (pi), introduction to trigonometric functions and to vectors Descriptive statistics: average values, variance, standard deviation Classic probability, probability theorems
Grades 11 and 12	Analytic geometry Conics Spatial geometry: planes, lines, polyhedra, pyramids, solids of rotation (cylinder, cone, sphere), areas and volumes of elementary solids Trigonometry: triangles, law of sines and law of cosines law Trigonometric functions: trigonometric equations and inequalities Exponential functions: exponential equations and inequalities Logarithmic functions, *e* (base of natural logarithms), logarithmic equations and inequalities Arithmetic and geometric sequences and series Mathematical induction Complex numbers: algebraic, geometric and trigonometric forms and representations; sums, products, and quotients; complex solutions of quadratic equations Combinatorics Statistics: regression and correlation Conditional probability, Bayes' theorem
Grade 13	Limit of a series Functions Limits: finite or infinite limits, limits of sums, products, quotients or composite functions, asymptotes Continuity on an interval Differentiation: numerical derivatives; the derivatives of common functions; derivatives of sums, products, quotients, and composite functions; applications of derivatives; the relationship between differentiability and continuity; the fundamental theorems of differential calculus; maxima and minima Integration: integration on an interval, the relationship between the definite integral and the area under a curve, the antiderivative of a function, applications of definite integrals Differential equations and applications, particularly in physics Analytic spatial geometry: coordinates, equations of planes, lines, spheres Probability: discrete and continuous random variables, probability distributions (Bernoulli, Poisson, normal), variance, standard deviation

The topics taught at each grade at *Istituti Tecnici–Settore Tecnologico* are essentially the same as those listed above, but have a more applicative orientation at Grades 11, 12 and 13. Also it should be noted that some topics of calculus (function limits, continuity, derivatives) are taught in

Grade 12, instead of Grade 13 as in *Liceo Scientifico*, while the integral calculus is taught in Grade 13 as in *Liceo Scientifico*. In these technical institutes, at Grades 12 and 13, many mathematical topics that serve specific technological applications are taught, such as partial derivatives, Fourier series, Taylor's formula, spherical trigonometry, etc.

In this type of high school, at the end of the five-year course, the study of mathematics helps students achieve the following learning outcomes:

- Mastery of formal language and demonstration procedures of mathematics
- Possession of the mathematical, statistical, and probability tools necessary for the understanding of scientific disciplines and the ability to work in the field of applied science
- Understanding of the place of mathematics in the history of science

Lebanon

At Grade 12, students receive a solid mathematical foundation with the aim of preparing them to pursue their studies as teachers, engineers, and researchers. The mathematics competencies students must have in each domain are provided in the table below.

Domain	Competencies
Mathematical Reasoning	Recognize the difference between a mathematical explanation and concrete or experimental evidence Make conjectures and discover means to test them Carry out proofs using various modes of reasoning Analyze and prove a statement of necessary and sufficient conditions Recognize a universal statement, a statement of existence, and a statement of uniqueness Evaluate a mathematical argument and criticize a proof Carry out an inductive proof
Problem Solving	Formulate a problem out of situations studied in mathematics, in other sciences, or encountered in real life Use various mathematical interpretations to represent the information given in a problem, figure out a convenient strategy to solve it, and take various approaches to make this strategy work using mathematical knowledge Discuss the validity of obtained solutions
Communication	Give a summary of a mathematical document Take notes on a mathematical lecture Critique a mathematical presentation Write a proof correctly
Spatial	Prove and apply the properties of solid figures and conics Characterize plane or space figures using vector notation Study geometric problems analytically Determine the effect of transformations on plane figures
Numbers and Algebra	Analyze the extensions of the sets of numbers (natural numbers, integers, rational numbers, real numbers, complex numbers) Study the properties of complex numbers and their use in geometry and trigonometry Generalize fundamental mathematical notions (set, relation, binary operation, and propositional calculus) Acquire an example of structure Develop mathematical tools for numerical calculations, and for solutions of systems of equations and inequalities
Calculus	Acquire the fundamental concepts of limit, continuity, and differentiability, and use them graphically Analyze the graphs of polynomial, rational, irrational, trigonometric, logarithmic, and exponential functions Integrate functions and solve simple differential equations
Statistics and Probability	Organize information and represent it graphically Study the characteristics of a statistical distribution of one variable Solve simple probability problems, mainly in discrete cases where the events are equally likely

Norway

TIMSS 2003 and PISA 2003 showed a decrease in Norwegian students' performance in mathematics and science in compulsory school compared with TIMSS 1995 and PISA 2000. This resulted in a broad discussion about how to improve the learning outcomes in Norway. A big effort was made to change the curriculum for all subjects in all 13 grades. There was an agreement nationally that something had to be done, and the new curriculum received support across all political parties in the parliament. It was called the Knowledge Promotion Reform, and was implemented in the autumn of 2006. The last cohort using the previous curriculum was in Grade 13 in the 2007–2008 school year, which means that these students were assessed in TIMSS Advanced 2008. Students assessed in TIMSS Advanced 2015 have been taught according to the 2006 curriculum.

In the present curriculum, two features stand out. First, the learning goals are formulated as competencies. Second, there are five basic skills (literacies) which are supposed to be used and developed in all subjects and at all levels: the ability to express oneself orally, the ability to read, the ability to express oneself in writing, the ability to use digital tools, and numeracy. Digital devices are supposed to be widely used in teaching, learning, and testing.

The following table indicates topics taught in the courses Mathematics R1 and Mathematics R2, normally taken in Grades 12 and 13, respectively.

Content Area	Topics
Geometry (R1 and R2)	Selected elements of Euclidean plane geometry, including geometric loci and similarity; constructions with compass and straightedge, and with geometry software; the intersection theorems for heights, angle bisectors, perpendicular bisectors and medians in a triangle; various proofs of Pythagoras' Theorem; vectors in the plane, with and without coordinates; application of vectors to determine lengths, angles, and parallelism and orthogonality of lines; vectors in space, with and without coordinates; application of scalar and vector products to determine distances, angles, areas and volumes; representation of lines, planes, and spheres by equations and in parametric form; calculation of lengths, angles and areas in bodies limited by planes and spheres
Algebra (R1 and R2)	Division and factorization of polynomials; logarithms; polynomial, rational, and logarithmic equations and inequalities; transformation and simplification of rational functions and other symbolic expressions with and without the use of digital aids; direct and contrapositive proof; proof by induction; number patterns; finite arithmetic series; finite and infinite geometric series; convergence
Functions (R1 and R2)	Limit, continuity and differentiability; derivatives of polynomial, exponential, and logarithmic functions; derivatives of sums, differences, products, and quotients of functions, and of composite functions; interpretation of functional behavior from the first and second derivatives; interpretation of derivatives in models of practical situations; drawing function graphs by hand and by digital tools; interpretation of a function's basic properties from its graph; horizontal and vertical asymptotes; vector functions with parameters; velocity and acceleration as derivatives of vector functions; trigonometric functions and equations; derivatives of trigonometric functions;

Content Area	Topics
Functions (R1 and R2) (Continued)	modeling periodic phenomena; definite and indefinite integrals; integration by substitution, by parts, and by partial fractions with linear denominators; interpretations of definite integrals in practical applications, and calculation of areas of plane regions and volumes of solids of revolution; mathematical modeling based on observed data
Combinatorics and Probability (R1 only)	Independence and conditional probability; Bayes' theorem for two events; ordered samples with and without replacement; unordered samples without replacement; applications to calculation of probabilities
Differential Equations (R2 only)	Modeling practical situations by differential equations; interpretation of solutions; linear first order and separable differential equations; second order homogenous differential equations; the use of Newton's second law and second order differential equations to describe free oscillations by periodic functions; application of digital tools to draw vector diagrams and integral curves

The previous curriculum for advanced mathematics covered quite a bit of statistics, including binomial, hypergeometric, and normal distributions, confidence intervals, and hypothesis testing. This was an important part of the curriculum in both of the advanced mathematics courses. The present curriculum has much less on statistics. The remaining parts are some combinatorics and probability taught in the first year of the advanced mathematics track (Mathematics R1). Another important change in the curriculum is that mathematical proof is emphasized more in the present curriculum than in the previous one. The new curriculum states that students shall "give an account of implication and equivalence, and implement direct and contrapositive proof" the first year (Mathematics R1) and "implement and give an account of proof by induction" the second year (Mathematics R2).

There have only been minor adjustments made to the curriculum after 2006. Both the new and the previous curricula emphasize the use of digital tools in mathematics. Under previous curricula, a liberal policy was developed to encourage and allow an extensive use of aids in all teaching and testing. Written notes and advanced calculators were normally allowed in local tests as well as in national written examinations. This has changed in the present curriculum. Every exam in mathematics is now divided into two parts. The first part is solved by pen and paper only and no aids are allowed. The second part, however, does not only allow the use of digital tools, but some are even required, like dynamic geometry programs. It is specifically stated that students in the second part of the exam shall have quite sophisticated electronical aids available.

Not all students have to take a national written exam in mathematics. About 40 percent of the first year (Mathematics R1) students are sampled, as are about 60 percent the second year (Mathematics R2) students. For the local oral exam, about 5 percent and 15 percent of the students in the respective courses are sampled for testing.

There is no national certification of teaching materials, such as textbooks, in Norway. The authors and publishers are free to decide the content of a textbook; the responsibility for covering the national curriculum rests on the school and the teacher.

Generally, one may say that the present curriculum emphasizes pure mathematics a little more than the previous one, across all levels. For instance, the present curriculum has a slightly stronger emphasis on algebra in compulsory school. Also, as has already been mentioned, formal proofs are now more emphasized than before in the advanced mathematics courses of upper-secondary school.

Portugal

Advanced Mathematics is a mandatory course for students in the upper-secondary Science and Technology and Socioeconomic Sciences academic tracks. The curriculum is divided into three main subjects: Probability and Combinatorics, Introduction to Differential Calculus II, and Trigonometry and Complex Numbers. The topics included in each main subject are listed below.

Main Subject	Topics
Probability and Combinatorics	Introduction to probability: random experiments; outcome spaces; events and operations with events; classical, frequency and axiomatic definitions of probability; conditional probability and independence of events Relative frequency and probability distributions: random variables and density functions for discrete variables; sample versus population means and standard-deviations; binomial probability distributions; normal distributions; histograms versus continuous probability density functions Combinatorics: enumerative combinatorics; permutations and combinations; Pascal's Triangle and Newton's Binomial expansion; the Binomial Theorem; applications of probability calculations
Introduction to Differential Calculus II	Exponential and logarithmic functions: analytical and graphical properties of exponential and logarithmic functions; rules for exponents and logarithms; modeling with exponential and logarithmic functions Limits theory: Heine's definition of the limit of a function and its properties; notable special limits; indeterminate forms of limits; asymptotes; continuity of functions, Bolzano-Cauchy's Theorem; numerical applications Differential calculus: Derivatives rules and applications; concavity and second derivatives; composite functions and their derivatives; properties of simple functions that can be determined by studying derivatives; optimization problems
Trigonometry and Complex Numbers	Trigonometry: intuitive study of the sine, cosine and tangent functions and their derivatives based on the unit circle; special limits of the sine function; use of trigonometry functions in modeling Complex numbers: introduction to complex numbers; the imaginary unit; algebraic form of and operations with complex numbers in this form; trigonometric form of complex numbers and operations with complex numbers in this form; geometric interpretation of operations with complex numbers; complex variables in the geometric plane

Russian Federation

High school programs for mathematics (Grades 10-11) are distinguished by the amount of the material being studied and the amount of instructional time. The Basic level program is designed for those students who plan to learn a profession that is not related to mathematics or plan to use mathematics as an auxiliary "tool" in their professional lives. The Profile level program provides sufficient depth of mathematics study to make it possible for students to enter a profession where mathematics is actively used. It includes a large amount of content and has higher requirements for its mastery. The mastery of this content makes it possible for students to continue to university-level studies in mathematical disciplines. Within the Profile level there is a subset of students in an even more intensive program taking six hours or more of mathematics lessons per week. The sample of students participating in the TIMSS Advanced 2015 Advanced Mathematics assessment included both Profile-level students and Intensive-level students. The results for students in the Intensive level were also reported separately as Russian Federation 6hr+.

The Profile level curriculum includes an explanation of the main goals of the program and provide for the organization and planning of mathematics courses, including:

- General characteristics of the profile course
- Teaching goals
- The number of lessons per week and per year
- General learning skills and activities
- Compulsory content and learning outcomes

The content of the Profile course is divided into two sections: Algebra and Calculus, and Geometry. The topics included in each section are listed below.

Content Areas in Algebra and Calculus	
Grade 10	
Polynomials	Transformation of polynomials, factorization; division of polynomials; Horner's method; roots of polynomials; Bezout's theorem; converting irrational expressions
Graphs of Functions	Complex functions; conversion of graphs; graphs of linear-fractional functions, asymptotes; graphs of functions which include a sign of a module (e.g., $y=\frac{2x-6}{\lvert 3-x\rvert}$ or $y=\sin\lvert x\rvert$) ; reciprocal functions and their graphs
Introduction to Calculus	Numerical sequences, limits of sequences, limits of functions, theorems on limits of functions; properties and continuity of elementary functions
Derivatives and their Applications	Geometric and physical meaning of the derivative, continuity and differentiability of functions, derivatives of sums, products, quotients, composites and exponential functions; second derivatives and higher order derivatives; application of derivatives to study functions; Lagrange's theorem and its consequences; drawing graphs of functions

Content Areas in Algebra and Calculus	
Trigonometric Functions	Trigonometric functions of numeric argument (sine, cosine, tangent and cotangent); trigonometric identities and their consequences; reduction formulas; identical transformation of trigonometric expressions; periodicity of trigonometric functions; properties, graphs, and derivatives of trigonometric functions
Grade 11	
Integral and Differential Equations	Indefinite integrals; definite integrals and their properties, numerical approximation of definite integrals, approximate computation; Newton-Leibniz formula; application of integrals for calculating areas, volumes, and lengths of arcs in physical problems; solutions of simple differential equations
Exponential and Logarithmic Functions	Properties and graphs of exponential functions; logarithms, definitions, and properties; identical transformations of exponential and logarithmic expressions; exponential and logarithmic equations, inequalities and systems of inequalities, types and methods of solution; derivatives of exponential functions; natural logarithms, radioactive decay
Complex Numbers	Algebraic form, arithmetic operations, conjugating complex numbers; solutions of quadratic equations with complex coefficients; the complex plane; trigonometric form of complex numbers, multiplication, division, and raising to power; De Moivre's formula; complex roots of polynomials; the Fundamental Theorem of Algebra
Elements of Combinatorics	Methods of mathematical induction; proofs of identities; factorials; the basic formulae of combinatorics; combinations and permutations; Binomial Theorem, Dirichlet's Principle
Elements of the Theory of Probability and Mathematical Statistics	Classic definition of probability, calculating probabilities using combinatorics; conditional probability, the rules of addition and multiplication of probabilities, independent events, Bernoulli distribution; mathematical expectation and variance; the concept of the law of large numbers and a normal distribution law; parent population and sample, levels of significance and reliability; evaluation of probability using frequency; the concept of statistical hypothesis testing
Equations, Inequalities, Systems	General methods and approaches for solving equations; irrational equations; generalized method of intervals for solving inequalities; systems of equations and inequalities, basic methods for solving systems of equations; application of graphs to solve equations, inequalities and systems; approximate methods for solving equations; equations, inequalities, and systems with parameters

Content Areas in Geometry	
Grade 10	
Axioms of Solid Geometry	
Parallel Lines and Planes	Mutual arrangement of lines and planes in space; theorems of parallelism of lines and planes
Perpendicularity of Lines and Planes	Theorems of dependences between parallelism and perpendicularity of lines and planes, the Theorem of the Three Perpendiculars; angles between straight lines and a plane
Coordinates and Vectors in a Space	Rectangular coordinate systems on a plane, the formula for distance between points, equations of straight lines and circumference; Cartesian coordinate system in a space, equations of straight lines and a plane; movements in a space and their properties (central symmetry, parallel translation, rotation), similarity in a space
Vectors in a Space	Decomposition of vectors into three non-coplanar vectors; scalar products; applications of coordinates and vectors to solve problems

Content Areas in Geometry	
Grade 11	
Polyhedrons	Concepts of polyhedrons, prisms, rectangular parallelepipeds, and pyramids; areas of faces and surfaces; sections; regular polyhedrons; dihedral angles
Solids of Revolution	Bodies and surfaces of revolution, cylinders, cones, axial sections of cylinders and cones; spheres and solid spheres, sections of solid spheres, equation of a sphere; inscribed and circumscribed cylinder, cone, sphere
Volumes of Bodies	Volumes of polyhedrons (prisms, pyramids) and solids of revolution (cylinder, cone, sphere, part of the sphere)
The Surface Areas of Solids of Revolution	Areas of spheres, surface areas of cylinders and cones

Learning outcomes are described in terms of what students should know and be able to do in each of these areas. Teachers have some discretion as to the introduction of optional topics.

Slovenia

In curricular documents for teachers and students, mathematics is presented as one of the basic subjects of general *gymnasia* in which students learn mathematics concepts and structures, critical thinking, and reasoning; develop creativity, formal knowledge and skills; recognize the practical usefulness of mathematics; gain mathematics knowledge and competencies needed for future mathematics studies as well as learning in other subjects and everyday life. The gymnasium mathematics course is compulsory and the same for all future university students, regardless of their area of study. The national curriculum for advanced mathematics is available in the form of printed and e-books containing general goals, contents and topics, expected student outcomes, and recommendations for teaching, including the incorporation of ICT, homework, and assessments into mathematics courses. In addition to the curriculum that is written for teachers' use, the expected standards, list of topics and examples of questions for basic and advanced level of the mathematics *matura* examination exist in printed and e-documents for students.

Contents and topics are given in the general order of teaching the advanced mathematics course through four years. For each topic, expected goals for students are followed by list of specified topics to be taught, expected hours of lessons needed for the content, and didactical recommendations about use of ICT. Included also are suggestions and guidelines for connecting the topics with material from other academic areas and how the topics could be presented and taught in these contexts. There are some topics classified as optional or as left to the teacher's discretion based on the teacher's expectations for students' achievement. The prescribed topics in each compulsory and elective content are listed below.

Content Area	Topics
Sets and Logic	Basics of logic; sets
Numbers	Number sets with whole, rational, real, and complex numbers (mathematical induction and the polar form of complex numbers are optional topics)
Algebraic Expressions	Equations and inequalities and their methods of solution (parametric equations are optional); powers and roots
Geometry	Lines, angles, circles and triangles in a plane and in space; sines and cosines; the areas of 2-D geometric shapes and the volumes of 3-D shapes and sections; Cartesian coordinate systems; vectors in a plane and in space, scalar product (vector product is optional)
Functions	Limits, continuity, inverse and composite functions; linear functions; solving systems of linear equations; quadratic, exponential, rational, logarithmic and trigonometric functions; conic sections
Sequences and Series	
Calculus	Differential calculations; integrals; applications of integrals
Probability and Statistics	Combinatorics

Expected outcomes are given by main topics as a list of content and procedural knowledge, provided in the table below. Procedural knowledge outcomes include general skills and processes linked to mathematical knowledge but transferable also to other areas.

Knowledge	Expected Outcomes
Content Knowledge	Calculate with numbers
	Use properties of sets
	Use logic in proofs
	Understand linear, power, root, quadratic, exponential, logarithmic, rational and trigonometric functions and calculate with them
	Draw graphs and use them in modeling
	Use Euclidean geometry and trigonometric functions in the context of Euclidean geometry; link Euclid geometry and vectors
	Use conic sections in problems
	Know and use arithmetic and geometric sequences and series, and apply them in financial mathematics and natural growth context
	Understand and use derivatives and determine tangents and simple extrema problems
	Know the meanings of indefinite and definite integrals; find indefinite integrals in simple situations, and use definite integrals for calculations of the area of a surface of revolution and volume of a solid of revolution
	Understand and use the fundamental principle of counting and other principles of combinatorics
	Know the classic definition of probability and calculate the probability of compound events
	Know statistical concepts, use them in other subject areas, and provide statistical analysis for a given problem
Procedural Knowledge	Abstract thinking
	Understanding of formal mathematical reasoning
	Analytical problem solving with different strategies
	Use of mathematics in everyday life (geometry, measurement, estimations, data analysis, interest expenses)
	Developing effective reading strategies for future learning
	Communicating mathematics in oral, written and other forms in the mother tongue and in one foreign language
	Designing and carrying out a research study and critically reporting findings
	Formulating research questions and hypotheses
	Thinking about necessary and sufficient conditions
	Using ICT and the Internet responsibly
	Making decisions and giving estimates of risks

Cross-curricular connections are provided as examples of activities that can link together knowledge from different subjects and mathematics.

Didactic recommendations describe the compulsory use of ICT in as many possible forms and activities as possible:

- To develop skills
- To reach new knowledge
- To help students with disabilities
- To help with calculations, statistics and in communication

All available digital devices (computers, tablets, graphic classroom boards, advanced calculators) and specialized software for learning mathematics (geometry simulations, symbolic calculations, drawing) are encouraged to be used for learning mathematics.

Homework is presented as the basic form of self-motivated learning and primary source for discussions in a class. It is said to help student attain better knowledge and may indirectly influence students' grades. Students should be assessed by at least four written tests and one oral examination in class per year. Other forms are also suggested (projects, research, group work) with the recommendation that students be giving enough opportunities to demonstrate their knowledge in different situations and are encouraged to develop responsibility for their own learning.

For the *matura*, students can decide whether to take the basic level or advanced level of the compulsory mathematics exam. The curriculum contents for both are the same, but required standards differ. The written test for the advanced level, in addition to compulsory items for the basic level, contains additional advanced level items. For oral examinations, the expected knowledge for the advanced level is specified in the *matura* standards (i.e. theoretical explanation of the definition of a limit versus the calculation of the limit only). Students receive grades from 1 to 5 for the *matura* exam at the basic level and from 1 to 8 at the advanced level. The sum of grades from all five *matura* subjects is used as a criterion for entrance to tertiary-level education programs with a limit on the number of new students.

Sweden

Four consecutive mathematics courses, Mathematics 1–4, comprise the mathematics curriculum covered by Swedish advanced mathematics students in upper-secondary school. In addition, students can choose to take additional mathematics courses. All courses are defined by a national curriculum including the goal of the subject, core content, and assessment criteria. These curricula describe learning objectives in short texts and teachers are expected to interpret the brief descriptions.

The curriculum dictates that mathematics courses should give students the opportunity to develop their ability to:

- Use and describe the meaning of mathematical concepts and their inter-relationships
- Employ procedures and solve standard tasks with and without tools
- Formulate, analyze and solve mathematical problems, and assess selected strategies, methods and results
- Interpret a realistic situation and design a mathematical model, as well as use and assess a model's properties and limitations
- Follow, apply, and assess mathematical reasoning
- Communicate mathematical thinking orally, in writing, and in action
- Relate mathematics to its importance and use in other subjects, in a professional, social and historical context

These competencies are the same for all courses, but the core content differs.

Algebra is introduced in compulsory school, and given a more comprehensive coverage in upper-secondary school. Early on in upper-secondary school the concept of linear inequality as well as algebraic and graphical methods for solving linear equations and inequalities, and exponential equations are introduced. Students later learn about logarithms. Students learn to solve different kinds of equations, including exponential, second degree polynomial and root equations, as well as systems of linear equations. The core content covers the concept of absolute values, and the concepts of polynomial and rational expressions, and generalization of the laws of arithmetic for dealing with these concepts. Furthermore, the number system is extended through the introduction of the concept of complex numbers in connection with solving second-degree equations. Mathematics 4 gives a more comprehensive coverage of different aspects of complex numbers.

In Geometry, the core content is mostly found in the first two mathematics courses. In Mathematics 1, students are introduced to the concepts of sine, cosine and tangent, as well as vectors and their representations. Students add and subtract vectors and do scalar multiplication. Geometry is used in order to illustrate the concepts of definition, theorem and proof. Students learn about the properties of the equation of a circle and are introduced to the unit circle in defining trigonometric concepts. In Mathematics 4, the core content contains a deeper coverage of trigonometry, for example methods for solving trigonometric equations.

Content relating to functions and calculus is found under the heading of Relationships and Change in all four mathematics courses. Students are taught about different kinds of functions and their properties. Calculus is added in Mathematics 3, starting with a brief introduction to continuous and discrete functions, as well as the concept of limits. Differentiation and use of the rules of differentiation for power and exponential functions, and also sums of functions, is described in the core content, as are algebraic and graphical methods for determining the value of the derivative of a function. Lessons should cover algebraic and graphical methods for solving extreme value problems using sign tables and second derivatives, and the relationship between the graph of a function and the first and second derivatives of a function. In Mathematics 4, the study of functions is expanded to include properties of trigonometric functions, logarithmic functions, compound functions and absolute values as functions. Lessons in differentiation and the use of the rules of differentiation for trigonometric, logarithmic, exponential and compound functions, and also the product and quotients of functions are included. In addition, students are expected to learn about algebraic and graphical methods for determining integrals with and without digital tools.

The core content also includes some arithmetic as well as probability and statistics, covered in the first two courses, and not as relevant to studies in advanced mathematics.

Problem solving is described as a core content in all four courses taken by advanced mathematics students in Sweden. Lessons cover strategies for mathematical problem solving including the use of digital media and tools, mathematical problems of importance in societal life and applications in other subjects, and mathematical problems related to the cultural history of mathematics.

United States

The United States does not have a uniform curriculum for advanced mathematics. For TIMSS Advanced 2015, students were sampled from courses identified as calculus using the definitions from the School Codes for the Exchange of Data (SCED) course classification system. The SCED courses included two College Board Advanced Placement (AP) courses (AB and BC), two International Baccalaureate (IB) Diploma Programme courses (IB Mathematics Standard Level and IB Mathematics High Level), and other courses implemented at the state, district, or school level. Descriptions of courses and their content in school catalogues were reviewed to determine course eligibility. As a result, the students assessed in TIMSS Advanced 2015 participated in varying curricula. The AP and IB courses have specific curricula that are taught to all students regardless of the state, district or school in which they take them.

In AP Calculus AB, the curriculum is broken into three major topic areas: functions, graphs, and limits; derivatives; and integrals. Under functions, graphs, and limits, the curriculum covers analysis of graphs, limits of functions (including one-sided limits), asymptotic and unbounded behavior, and continuity as a property of functions. Under derivatives, the curriculum covers the concept of a derivative, derivative at a point, derivative as a function, second derivatives, and applications and computation of derivatives. Under integrals, the curriculum covers interpretations and properties of definite integrals, application of integrals, Fundamental Theorem of Calculus, techniques and application of antidifferentiation, and numerical approximation of definite integrals.

AP Calculus BC has a similar curriculum as AP Calculus AB, and covers all of the topics of AP Calculus AB, with additional material. Under functions, graphs, and limits, the AP Calculus BC curriculum additionally covers parametric, polar, and vector functions. AP Calculus BC also has a fourth major topic area: polynomial approximations and series. This topic covers the concept of series, series of constants, and Taylor series.

IB Mathematics Standard Level (SL) has a core curriculum that covers algebra, functions and equations, circular functions and trigonometry, matrices, vectors, statistics and probability, and calculus (differential and integral). The curriculum also requires all students to complete a portfolio of two individual pieces of work, based on mathematical investigation and mathematical modeling. IB Mathematics Higher Level (HL) has the same core curriculum and portfolio requirements as IB Mathematics SL, but additionally requires 40 hours of instruction in one of the following topics: statistics and probability, sets, relations and groups, series and differential equations, or discrete mathematics.

The other courses that students were sampled from are "Calculus and Analytic Geometry" and "Calculus", with course curricula varying by state, district, or school.