

Appendix 14A: TIMSS Advanced 2015 Advanced Mathematics Item Descriptions Developed During the TIMSS Advanced 2015 Benchmarking

Items at Intermediate International Benchmark (475)

Algebra

M2_01	Recognizes the graph of the absolute value of a function given the graph of the function
M3_01	Determines which term has a given value in a geometric sequence
M3_04	Analyzes steps in a given solution of a simple logarithmic equation and identifies an error
M7_02	Computes the value of a composite function at a given value
M8_01	Identifies the expression that results from the composite of a function with itself
M9_02	Evaluates an exponential expression with three unknowns given three possible values for each unknown

Calculus

M1_05	Differentiates an exponential function where the exponent is a simple polynomial
M3_06	Analyzes the graph of a function to determine the sign of its derivative
M4_05	Computes the limit of an exponential function
M6_05	Integrates the sum of an exponential function and a monomial
M8_05	Determines the limit of a rational function in terms of an unknown constant

Geometry

M1_08	Calculates the difference between vectors in coordinate form
M2_11	Finds the length of a diagonal of a regular hexagon of given side length
M3_09	Evaluates the shortest path between opposite vertices on the surface of a cube
M3_10	Solves a word problem about height given the distance and angle of elevation
M6_09	Recognizes a diagram of the sum of three vectors
M9_09	Identifies the length of a side of an isosceles triangle using properties of a similar triangle

Items at High International Benchmark (550)

Algebra

M1_01	Rationalizes the denominator of an expression
M1_03	Determines when a rational function with numerator and denominator in factored form is negative
M2_02	Indicates whether factored polynomials satisfy two given conditions (2 of 2 points)
M2_05	Determines the values of two constants in a rational expression given its graph with two specified points
M3_02	Solves a word problem about the number of permutations
M3_03	Solves a word problem involving dimensions of two cylindrical containers given their volumes (2 of 2 points)
M4_01	Simplifies an expression with log base 10 in the exponent
M4_04	Determines the values of two constants in a rational function given its asymptotes
M5_02	Identifies two constants in a rational function given two points on its graph
M5_05A	Solves a word problem by finding the distance between the points at which a parabola intersects the x-axis
M6_01	Recognizes the graph that could represent a function and its inverse
M6_02	Identifies the solution of a quadratic inequality
M6_03	Solves an exponential decay equation for the time at which a specified amount of substance remains
M7_01	Determines the interval on which a given rational function is greater than the square of that function
M7_03	Multiplies complex numbers
M7_04	Determines the domain of a logarithm of a rational function
M8_02	Finds the value of a particular term of an arithmetic sequence
M8_03	Uses the initial value of a fractional expression with three unknowns to evaluate the expression after the unknowns are divided by multiples of 2
M9_01	Identifies an increasing function defined for all real numbers
M9_04	Determines the value of an unknown in a logarithmic equation given its two solutions

Calculus

M2_07	Identifies the graph of a function that satisfies given conditions for the first and second derivatives
M2_08	Determines the limit of a rational function in terms of an unknown constant

M3_05	Finds the second derivative of a rational function (2 of 2 points)
M4_06	Identifies the derivative of a composite trigonometric function
M4_07	Identifies the graph of a function given the graph of its first derivative
M4_08	Identifies the values of a definite integral with an unknown upper bound
M6_07	Identifies the local maximum of a function given intervals on which its first and second derivatives are positive, negative, and zero
M6_08	Sketches the graph of a function on a specified interval with three given properties (2 of 2 points)
M7_05	Uses the additivity of intervals to identify the value of a definite integral
M7_06	Determines the derivative of the product of a monomial and an unspecified function
M9_05	Identifies a true statement about discontinuity and non-differentiability for a graph of a piecewise function
M9_06	Recognizes the graph of the derivative of a curvilinear, discontinuous function
Geometry	
M2_10	Determines the value of a trigonometric function given the value of a related function
M4_09	Determines the ratio of the squares of two sides of a scalene triangle given two of its angles
M4_10	Finds the maximum value of a trigonometric function and a value of the independent variable at which it occurs (2 of 2 points)
M4_11	Proves that a quadrilateral with given coordinates of its vertices is a parallelogram
M5_08	Identifies coordinates of the fourth vertex of a parallelogram when three vertices are given
M7_10	Recognizes the description of a sine graph transformation
M8_09	Identifies a vector that is perpendicular to a given vector in a coordinate system
M8_10	Determines the lengths of two sides of a triangle given its area, the sum of the lengths of the two sides, and the angle included between them
M9_10	Determines the coordinates of line segment endpoints given the midpoint

Items at Advanced Benchmark (625)

Algebra

M1_02	Calculates the cube of a complex number given in trigonometric form
M2_03	Finds the sum of the first 100 terms of an alternating series at a given value of x
M2_04	Determines the sum of an infinite alternating geometric series

M4_02	Determines the intersection of two functions in terms of an unknown, non-zero coefficient (2 of 2 points)
M4_03	Determines the value when one cost becomes less than another and explains whether increasing the initial costs will change the value (2 of 2 points)
M5_01	Given the first three terms, calculates the sum of an infinite geometric series
M5_03	Solves a logarithmic equation (2 of 2 points)
M5_04	Given one imaginary root, identifies the constant term of a third-degree polynomial with known coefficients
M6_04	Determines the coefficient of the linear term and the constant of a quadratic equation given its solution
M8_04	Determines the amount of time that a ball is at or above a specified height given the quadratic function for its height (2 of 2 points)
M9_03	Determines the values of an unknown coefficient for which the graph of a parabola lies above the x-axis
Calculus	
M1_06	Maximizes the volume of a cylinder given a relationship between its height and diameter
M2_06	Identifies the value of a definite integral from areas shown on a graph
M2_09	Explains whether a given piecewise function is continuous at a given value
M3_08	Calculates the area between the graphs of a linear and a quadratic function (2 of 2 points)
M5_06	Given the graph of the derivative of a function, determines the x-values of the maximum point and the point of inflection of the function (2 of 2 points)
M6_06	Explains whether a right-hand limit and a left-hand limit of a function are equal
M7_07	Maximizes the area of a rectangle with constraint on the sum of three sides and explains why the solution gives the maximum area (2 of 2 points)
M7_08	Identifies specific properties of the first and second derivatives of a function given its graph
M7_09	Determines the limit of a rational function in terms of an unknown constant
M8_06	Indicates whether statements about the continuity or differentiability of a function with given conditions are true (2 of 2 points)
M8_07	Solves a multi-step word problem by maximizing the profit given a quadratic cost function and the linear income function (2 of 2 points)
M9_08	Determines the equation of a line parallel to a tangent line of a given function at a specified point (2 of 2 points)
M9_08	Indicates whether statements about a function are true given a graph of the derivative (2 of 2 points)

Geometry

M1_07	Identifies the equation of a line through a given point and perpendicular to a given line
M1_10	Uses vector sums and differences to express a relationship among three vectors shown in a figure
M2_12	Determines the length of a line segment in a problem involving similar right triangles
M3_11	Uses properties of vectors to analyze equivalence of conditions involving the sum and difference of two vectors
M5_09	Compares amplitudes and periods of sine functions
M6_10	Justifies a statement regarding the length of the radius of a circle drawn on a square grid (2 of 2 points)
M6_11	Identifies the parameter of a sine function given the graphs of a function and its transformation
M7_11	Solves a word problem involving concentric circles and areas of sectors (2 of 2 points)
M7_12	Explains why the sum of a sine and a cosine function does not exceed a specified value
M8_11	Determines a diagonal length of a rhombus in terms of the length of a side given the ratio between the obtuse and acute angles
M9_11	Identifies the parameters of a cosine function used to model data presented in a graph
M9_12	Proves that a trigonometric relation holds for a triangle with specified angle and side measures

Items Above the Advanced International Benchmark (625)

Calculus

M3_07	Solves a multi-step word problem by maximizing the profit given a quadratic cost function and the unit selling price
M5_05B	Solves a multi-step word problem by calculating the area between two intersecting parabolas
M5_07	Determines the vertical line that divides a specified area between a parabola and the x-axis into equal parts
M8_08	Shows a process for integrating the product of a linear and a trigonometric function

Geometry

M1_09	Given two points, identifies an equation that represents the set of all points twice as far from one of the given points as from the other
M5_10	Calculates the two possible lengths of a side of a triangle given an angle and the lengths of two sides that do not include the angle (2 of 2 points)
M6_12	Proves the equality of sines of supplementary angles