

# IEA's TIMSS 2003 International Report on Achievement in the <br> Mathematics Cognitive Domains 

Findings from a Developmental Project
© 2005 International Association for the Evaluation of Educational Achievement (IEA)

TIMSS 2003 International Mathematics Report / by Ina V.S. Mullis, Michael O. Martin, Pierre Foy.

Publisher:
TIMSS \& PIRLS International Study Center,
Lynch School of Education, Boston College
Library of Congress Catalog Card Number: 2005900696
ISBN: 1-889938-38-6
For more information about TIMSS contact:
TIMSS \& PIRLS International Study Center
Lynch School of Education
Manresa House
Boston College
Chestnut Hill, MA 02467
United States
tel: $+1-617-552-1600$
fax: +1-617-552-1203
E-mail: timss@bc.edu
URL: timss.bc.edu

Boston College is an equal opportunity, affirmative action employer.
Printed and bound in the United States.

## Contents

## 3 Chapter 1

The Developmental Project to Report TIMSS 2003 Mathematics Achievement in Cognitive Domains

3 Overview of TIMSS
3 The TIMSS 2003 Assessment Frameworks and International Reports

5 History of the Developmental Project
6 Mapping the TIMSS 2003 Mathematics Items to Cognitive Domains

9 The Scaling Methodology
10 Summary of Overall Mathematics Achievement Nationally and by Gender for the TIMSS 2003 Countries
12 Exhibit 1.1 Distribution of Mathematics Achievement Overall and by Gender

## 15 Chapter 2

Mathematics Achievement in the Cognitive Domains at the Fourth and Eighth Grades

15 Knowing Facts, Procedures, and Concepts
18 Exhibit 2.1 Distribution of Mathematics Achievement for Knowing Cognitive Domain
20 Exhibit 2.2 Multiple Comparisons of Average Mathematics Achievement for Knowing Cognitive Domain

23 Applying Knowledge and Conceptual Understanding
24 Exhibit 2.3 Distribution of Mathematics Achievement for Applying Cognitive Domain
26 Exhibit 2.4 Multiple Comparisons of Average Mathematics Achievement for Applying Cognitive Domain

29 Reasoning
30 Exhibit 2.5 Distribution of Mathematics Achievement for Reasoning Cognitive Domain
32 Exhibit 2.6 Multiple Comparisons of Average Mathematics Achievement for Reasoning Cognitive Domain

36 Overview Across Domains

## Contents

## 41 Chapter 3

Achievement by Gender in the Mathematics Cognitive Domains at the Fourth and Eighth Grades

42 Gender Differences in the Knowing Cognitive Domain

## 43 Exhibit 3.1 Average Mathematics Achievement by Gender for Knowing Cognitive Domain

45 Gender Differences in the Applying Cognitive Domain
46 Exhibit 3.2 Average Mathematics Achievement by Gender for Applying Cognitive Domain

48 Gender Differences in the Reasoning Cognitive Domain
49 Exhibit 3.3 Average Mathematics Achievement by Gender for Reasoning Cognitive Domain

## 53 Chapter 4

Country by Country Profiles of Achievement in the Mathematics Cognitive Domains

53 Profiles of Achievement
54 Relative Strengths and Weaknesses in the Knowing Domain
55 Exhibit 4.1 Profiles of Within-Country Relative Performance in Mathematics Cognitive Domains

60 Relative Strengths and Weaknesses in the Applying Domain
60 Relative Strengths and Weaknesses in the Reasoning Domain
61 International Achievement Across the Cognitive Domains
63 References
65 Appendix A
Mathematics Cognitive Domains Framework: TIMSS 2003 Developmental Project Fourth and Eighth Grades

65 Knowing Facts, Procedures, and Concepts
67 Applying Knowledge and Understanding
69 Reasoning

## 73 Appendix B

Overview of Procedures TIMSS 2003 Developmental Project

73 Process for Establishing the Mathematics Cognitive Domains for Scaling and Reporting

## 75 Characteristics of Items Within Cognitive Domains

76 Exhibit B. 1 Characteristics of Items Within Cognitive Domains

## 79 Constructing Achievement Scales in the Mathematics Cognitive Domains

## 80 Item Calibration

81 Evaluating the Fit of the IRT Models
81 Generating IRT Proficiency Scores
82 Reliability
83 Exhibit B. 2 Reliabilities of Overall Mathematics and Cognitive Domains

## 85 Correlations

86 Exhibit B. 3 Correlations of Mathematics Cognitive Domains with Overall Mathematics

88 Exhibit B. 4 Correlations of Mathematics Cognitive Domains

## 93 Appendix C

Coverage of TIMSS 2003 Target Populations and Participation Rates
94 Exhibit C. 1 Coverage of TIMSS 2003 Target Populations
96 Exhibit C. 2 Participation Rates (Weighted)

## 99 Appendix D <br> Percentiles and Standard Deviations of Mathematics Achievement in the Cognitive Domains

100 Exhibit D. 1 Percentiles of Achievement in Knowing Cognitive Domain
102 Exhibit D. 2 Percentiles of Achievement in Applying Cognitive Domain
104 Exhibit D. 3 Percentiles of Achievement in Reasoning Cognitive Domain
106 Exhibit D. 4 Standard Deviations of Achievement in Knowing Cognitive Domain
108 Exhibit D. 5 Standard Deviations of Achievement in Applying Cognitive Domain
110 Exhibit D. 6 Standard Deviations of Achievement in Reasoning Cognitive Domain


## Chapter 1

The Developmental Project to Report TIMSS 2003 Mathematics Achievement in Cognitive Domains

## Overview of TIMSS

TIMSS 2003 is the third and most recently completed round of IEA's Trends in International Mathematics and Science Study, a very ambitious series of international assessments carried out in countries around the world to measure trends in mathematics and science learning at the fourth and eighth grades. Conducted first in 1995 and then again in 1999, the regular four-year cycle of TIMSS provides countries with an unprecedented opportunity to obtain comparative information about their students' achievement in mathematics and science. Forty-nine countries participated in TIMSS 2003, with 23 having participated in all three assessments and another 14 having participated in two rounds. In developing the instruments and procedures for TIMSS 2007, IEA is currently working with more than 60 countries.

## The TIMSS 2003 Assessment Frameworks and International Reports

For TIMSS 2003, the frameworks underlying the mathematics and science assessments and questionnaires were updated through a major effort. In particular, the mathematics and science frameworks were organized along two dimensions - content domains and cognitive domains. With additional financial support from the US National

Science Foundation and the US National Center for Education Statistics, IEA's TIMSS \& PIRLS International Study Center (ISC) worked with the participating countries to describe in detail the mathematics and science content to be assessed and to update the learning outcomes related to particular cognitive domains. The updated frameworks were published in the TIMSS Assessment Frameworks and Specifications 2003, 2nd Edition (Mullis, Martin, Smith, Garden, Gregory, Gonzalez, Chrostowski, and O'Connor, 2003).

For mathematics, the five content domains were number, algebra (called patterns and relationships at fourth grade), measurement, geometry, and data. Each content domain described the topic areas to be assessed within that domain, and each topic area was elaborated with objectives specific to the eighth and fourth grades. Four cognitive domains were described - Knowing Facts and Procedures, Using Concepts, Solving Routine Problems, and Reasoning - together with the skills and abilities making up each domain.

Developing the TIMSS 2003 tests was a cooperative venture involving all of the National Research Coordinators (NRCs), including field-testing the items with representative samples of students. The NRCs and the Science and Mathematics Item Review Committee (SMIRC) had several opportunities to review the items and scoring criteria. The resulting TIMSS 2003 mathematics tests contained 194 items at the eighth grade and 161 items at the fourth grade.

The international mathematics results from TIMSS 2003 were initially reported in the TIMSS 2003 International Mathematics Report: Findings from IEA's Trends in International Mathematics and Science Study at the Fourth and Eighth Grades (Mullis, Martin, Gonzalezand Chrostowski, 2004). This report contained overall mathematics achievement results for the participating countries as well as achievement in major content domains - number, algebra, measurement, geometry, and data. It also contained a rich array of information about the school and home contexts for learning mathematics including country-level information
collected from the NRCs and considerable data from student, teacher, and school questionnaires.

## History of the Developmental Project

Since the first round of TIMSS in 1995, IEA's TIMSS \& PIRLS ISC has reported on students' mathematics achievement in content domains (e.g., algebra, geometry) and, as noted above, TIMSS 2003 was no exception. The TIMSS content domains are fairly consistently found in the curricula of the participating countries and the results provide an indication of curriculum areas on which students perform relatively better of worse, both within and across countries. For example, TIMSS 1995, 1999, and 2003 have shown that, on average, eighth-grade students in the United States perform relatively poorly on geometry items and relatively well on data items. For policymakers and educators, such information can prove useful in discussions about the curricular foci and overall learning goals of students across the country.

Developing reliable and valid achievement scales for cognitive domains can be challenging, since the differences among students across and within countries in their mathematics knowledge and problemsolving skills make it difficult to know which cognitive abilities students are using to solve a given mathematics item. Nevertheless, considerable work has been done in this area by national and international assessments, including IEA's Progress in International Reading Literacy Study (PIRLS), the OECD's Programme for International Student Assessment (PISA), and the US National Assessment of Educational Progress (NAEP). For example, for the 2004 IEA research conference in Cyprus, the TIMSS \& PIRLS ISC reported international achievement in the processes of reading comprehension (Mullis, Martin, and Gonzalez, 2004) and PIRLS 2006 will institute achievement scales based on processes of comprehension.

Consistent with the growing practice of reporting achievement in various cognitive areas, countries participating in TIMSS also have
expressed a need for comparative information about how students perform in the cognitive domains. To provide enhanced information from TIMSS 2003 and facilitate planning for TIMSS 2007, a number of participating countries supported a developmental project for IEA's TIMSS \& PIRLS International Study Center to examine mathematics achievement by cognitive domains. Although focusing on mathematics as the first step, if successful the project was intended also to serve as a roadmap for achieving similar goals in science.

Led by the United States, with funding also provided by Chinese Taipei, Cyprus, New Zealand, Norway, Ontario, Quebec, Singapore, and Sweden, the developmental project involved several major activities. Prior to preparing this report of the results of the development study, IEA's TIMSS \& PIRLS ISC first convened an international meeting of experts in mathematics and mathematics education to confirm the mapping of TIMSS 2003 mathematics items to cognitive domains. Then, IEA's TIMSS \& PIRLS ISC conducted the various phases of the analytic work necessary to create the cognitive domain scale scores.

## Mapping the TIMSS 2003 Mathematics Items to Cognitive Domains

The developmental project began with a special meeting of mathematics experts held in February 2005 in Amsterdam, with the purpose of examining the classification of items according to the cognitive domains articulated in the TIMSS 2003 mathematics framework. The 10 participants (see Appendix B) expressed great enthusiasm for the meeting goal - facilitating TIMSS reporting according to cognitive domains. Nevertheless, all members expressed reservations about using the cognitive domains as they stood.

In developing the TIMSS 2003 Assessment Framework for Mathematics, there were no plans to scale and report results by the cognitive domains. In updating the cognitive domains and the learning outcomes related to them, the major goal was to encourage item writers to be as creative as possible and develop items across a variety of cognitive skills and abilities. Although this approach appeared to be
viable at the time, and most likely improved the quality of the items for TIMSS 2003, it did lead to some overlap across the four cognitive domains. For example, as demonstrated in assessment items, it was sometimes difficult to distinguish between "knowing facts and procedures" and "using concepts." This overlap made assigning items according to the four original categories very difficult for the members of the expert group. As a result, the expert group worked to use the existing framework as a basis for developing mutually exclusive cognitive domains for reporting the TIMSS 2003 results. The process was an iterative one involving independent classification of items and discussion. In classifying items, the expert group followed the guidelines of classifying items according to the cognitive process they thought most students would use.

Based on this process and final confirmatory rounds of classifying the TIMSS 2003 fourth- and eighth-grade items, the experts felt comfortable with three cognitive domains:

- Knowing Facts, Procedures, and Concepts,
- Applying Knowledge and Understanding,
- Reasoning.

The first domain, knowing facts, procedures, and concepts, covers what the student needs to know, while the second, applying knowledge and conceptual understanding, focuses on the ability of the student to apply what he or she knows to solve routine problems or answer questions. The third domain, reasoning, goes beyond the solution of routine problems to encompass unfamiliar situations, complex contexts, and multi-step problems.

Even though all the individuals who participated in the Amsterdam Cognitive Domains meeting felt that great progress had been made in establishing reliable and valid classifications for analysis and reporting, several additional confirmatory steps were taken. First, a second expert review was conducted as part of the first TIMSS 2007 SMIRC meeting held in April 2005. The SMIRC endorsed the work
accomplished at the special Mathematics Cognitive Domains meeting and worked toward refining the classifications and their descriptions to better reflect the essence of the three cognitive domains. This resulted in an excellent foundation for scaling the TIMSS 2003 achievement data by cognitive domains.

Also, IEA's TIMSS \& PIRLS ISC examined the distribution of the items within the three cognitive domains by item type, content domain, and difficulty to ensure that there was sufficient coverage of each of the newly defined domains. As described in Appendix B (and summarized in Exhibit B.1), there was a substantial number of items in each domain: 65 in knowing, 93 in applying, and 36 in reasoning at eighth grade; and 58 in knowing, 63 in applying, and 38 in reasoning at fourth grade. Within each domain, there was a good spread of item type (constructed-response or multiple-choice) at both grades, although as might be expected, relatively more of the knowing items were multiple choice and relatively more reasoning items constructed response. There also was a good spread of items across content domains within each of the three cognitive domains, although there was some unevenness in some areas. For example, it would have been preferable to have a higher proportion of number items in the reasoning domain at the eighth grade, and a higher proportion of patterns and relationship items in the knowing domain and measurement items in the reasoning domain at fourth grade. For TIMSS 2007, an effort has been made to address these issues in the assessment frameworks. Finally, there was a good range of item difficulty within each of the cognitive domains, with reasoning items most difficult, on average, as would be anticipated.

The Mathematics Cognitive Domains Framework for the TIMSS 2003 Development Project that was used as the basis of this report is found in Appendix A. It should be noted that this framework was further reviewed by the TIMSS 2007 National Research Coordinators at their second meeting in Amsterdam, June 2005, resulting in
further refinements for TIMSS 2007 as published in the TIMSS 2007 Assessment Frameworks (Mullis, Martin, Ruddock, O'Sullivan, Arora, and Erberber, 2005).

## The Scaling Methodology

The methodology used to create the mathematics cognitive domain scales was identical to that used to report mathematics achievement results and achievement in the mathematics content domains in the TIMSS 2003 International Reports (Mullis, Martin, Gonzalez $\mathcal{E}$ Chrostowski, 2004). TIMSS 2003 relied on item response theory scaling (IRT) to describe student achievement in mathematics overall, in the content domains, and in the cognitive domains. TIMSS created separate scales for mathematics overall, for each content domain, and for each cognitive domain at both fourth and eighth grades. The metric for the TIMSS overall mathematics scale was established originally in TIMSS 1995, with a mean of 500 and standard deviation of 100 across the countries participating in that first TIMSS assessment. This was done separately for fourth and eighth grades. To provide a mechanism for measuring changes in student achievement over time, the data from the TIMSS assessments in 1999 (eighth grade only) and 2003 (both grades) were linked to this scale. The international average score for the eighth-grade countries in 2003 was 467 , and for the fourth-grade countries, 495. To facilitate comparisons across cognitive domains and with overall mathematics, and following the procedure used for the mathematics content scales in 2003, the three cognitive domain scales were set to have the same mean and standard deviation as the overall mathematics scales, i.e., a mean of 467 and standard deviation of 100 at the eighth grade, and a mean of 495 and standard deviation of 100 at the fourth grade. The methodology is summarized in Appendix B and is described in detail in the TIMSS 2003 Technical Report (Martin, Mullis \& Chrostowski, 2004).

## Summary of Overall Mathematics Achievement Nationally and by Gender for the TIMSS 2003 Countries

To provide a context for considering mathematics achievement at the fourth and eighth grades in the cognitive domains, the first page of Exhibit 1.1 presents mathematics achievement for all students and separately by gender for the 46 countries and four benchmarking entities that participated at the eighth grade in TIMSS 2003 and the second page presents mathematics achievement in the same way for the 25 countries and three benchmarking entities that participated at the fourth grade. ${ }^{1}$ At each grade, countries are shown in decreasing order of average (mean) scale score, together with an indication of whether the country average was significantly higher or lower than the international average. ${ }^{2}$ It should be noted that the results for the eighth and fourth grades are not directly comparable. ${ }^{3}$

To recap the overall mathematics achievement results, reported in full in the TIMSS 2003 International Mathematics Report, Singapore was the highest-performing country at both the fourth and eighth grades. At the eighth grade, the Republic of Korea, Hong Kong SAR, and Chinese Taipei outperformed all the other countries except Singapore. Japan also performed very well, as did Belgium (Flemish), the Netherlands, Estonia, and Hungary. At the fourth grade, in addition to Singapore, Hong Kong SAR, Japan, and Chinese Taipei also had higher achievement than the rest of the countries as did Belgium (Flemish). Belgium (Flemish), however, was outperformed by the Asian countries.

To aid in interpretation, Exhibit 1.1 also includes the years of formal schooling and average age of the students in each country. At the eighth grade, the aim was that the students assessed would have had eight years of formal schooling. Similarly, at the fourth grade, the aim was to assess students having had four years of formal schooling. This was the case for most participating countries, however, as shown in the TIMSS 2003 International Mathematics Report, the TIMSS 2003 countries had different policies about the age at which students begin formal schooling and about promotion and retention from grade

2 The international average of 467 at the eighth grade was obtained by averaging across the mean scores for each of the 46 participating countries. The mean scores for the four benchmarking participants were not included in calculating the average. Even though England worked diligently to meet the TIMSS sampling requirements and adjustments were made to make the results representative, it did not meet the school participation rates as specified in the guidelines and consequently its results are shown below a line. At the fourth grade, the international average of 495 was obtained by averaging across the mean scores for the 25 participating countries.
3 While the scales for the two grades are expressed in the same numerical units, they are not directly comparable in terms of being able to say how much achievement or learning at one grade equals how much achievement or learning at the other grade. Comparisons only can be made in terms of relative performance. Since the TIMSS scales were developed using IRT technology, like all such scales, the eighth- and fourth-grade scales cannot be described in absolute terms.
to grade. Thus, even though TIMSS devoted considerable effort to maximizing comparability across the grades tested there was some variation. Most notably, in the eighth-grade population, students in Norway, most of Slovenia, and parts of the Russian Federation had fewer years of formal schooling than their counterparts in other countries, while those in England, Scotland, New Zealand, and parts of Australia had more years of schooling. In the fourth-grade population, some students in Slovenia and parts of the Russian Federation had only three years of formal schooling, and students in England and Scotland as well as some in Australia and New Zealand had five years. Also, equivalence of chronological age does not necessarily mean that students have received the same number of year of formal schooling or studied the same curriculum. At the eighth grade, students were on average between 14 and 15 years old, but the range of policies and situations in the participating countries led to considerable variation. At the fourth grade, students in most countries were on average between 10 and 11 years old.

As can be seen in the right-hand portion of both pages of Exhibit 1.1, at both the eighth and fourth grades, the difference in overall mathematics performance by gender was negligible in many countries. The situation did vary by country, however. At the eighth grade, girls had significantly higher achievement in Singapore, Armenia, Serbia, Moldova, Cyprus, Macedonia, Jordan, Bahrain, and the Philippines. Boys had significantly higher achievement than girls in Belgium (Flemish), Hungary, the United States, Italy, Lebanon, Tunisia, Chile, Morocco, Ghana, the US state of Indiana, and the Canadian province of Quebec. At the fourth grade, girls had significantly higher average mathematics achievement in Singapore, Moldova, Armenia, and the Philippines. Boys had higher average achievement in the Netherlands, the United States, Cyprus, Italy, Scotland, and in the two Canadian provinces.

## Exhibit 1.1: Distribution of Mathematics Achievement Overall and by Gender



* Represents years of schooling counting from the first year of ISCED Level 1
$\dagger$ Met guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
$\ddagger \quad$ Nearly satisfied guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
ま Did not satisfy guidelines for sample participation rates (see Exhibit C.2).

1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1)
2 National Defined Population covers less than 90\% of National Desired Population (see Exhibit C.1).

- Korea tested the same cohort of students as other countries, but later in 2003, at the beginning of the next school year.
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.


## Exhibit 1.1: Distribution of Mathematics Achievement Overall and by Gender <br> mathematics Grade <br> 

| Countries | Average <br> Scale Score |  | Years of <br> Schooling* |  |
| :--- | :--- | :--- | :--- | :---: |
| Singapore | $\mathbf{O}$ | $594(5.6)$ | Average |  |
| Age |  |  |  |  |


| Girls <br> Average Scale Score |  | Boys Average Scale Score |  | Difference <br> (Absolute Value) |
| :---: | :---: | :---: | :---: | :---: |
| 599 (5.5) | 0 | 590 (6.2) |  | 8 (3.9) |
| 575 (3.4) |  | 575 (3.4) |  | 0 (2.3) |
| 563 (1.8) |  | 566 (2.1) |  | 4 (2.3) |
| 564 (1.7) |  | 564 (2.1) |  | 1 (1.7) |
| 549 (1.8) |  | 552 (2.5) |  | 2 (2.5) |
| 537 (2.7) |  | 543 (2.2) | 0 | 6 (2.4) |
| 536 (2.9) |  | 536 (3.5) |  | 1 (2.9) |
| 535 (3.5) |  | 536 (3.2) |  | 1 (2.8) |
| 530 (5.4) |  | 534 (4.7) |  | 4 (3.5) |
| 530 (3.9) |  | 532 (4.5) |  | 2 (4.0) |
| 527 (3.8) |  | 530 (3.3) |  | 3 (3.4) |
| 514 (2.4) |  | 522 (2.7) | 0 | 8 (1.6) |
| 505 (2.7) |  | 514 (2.9) | 0 | 9 (2.8) |
| 510 (5.2) | 0 | 499 (5.1) |  | 11 (3.5) |
| 498 (4.1) |  | 507 (3.7) | 0 | 9 (2.6) |
| 497 (4.5) |  | 500 (4.3) |  | 3 (4.0) |
| 495 (0.8) |  | 496 (0.8) |  | 1 (0.7) |
| 493 (2.7) |  | 494 (2.4) |  | 0 (2.9) |
| 485 (3.2) |  | 496 (4.4) | 0 | 11 (4.1) |
| 477 (3.0) |  | 481 (3.5) |  | 5 (3.8) |
| 462 (3.7) | 0 | 450 (3.8) |  | 12 (2.9) |
| 449 (2.7) |  | 454 (2.7) |  | 5 (2.8) |
| 394 (6.5) |  | 386 (5.5) |  | 8 (8.8) |
| 364 (9.2) | 0 | 352 (7.0) |  | 12 (4.6) |
| 344 (6.1) |  | 350 (5.1) |  | 6 (4.7) |
| 342 (5.0) |  | 337 (4.9) |  | 5 (2.8) |
| 532 (3.1) |  | 534 (3.4) |  | 2 (3.3) |
| 505 (3.6) |  | 517 (4.7) | 0 | 11 (3.7) |
| 502 (2.7) |  | 509 (2.8) | 0 | 7 (2.7) |

( Country average significantly higher than international average
(7) Country average significantly lower than international average

[^0]の Norway: 4 years of formal schooling, but First Grade is called "First grade/Preschool.
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.


## Chapter 2

## Mathematics Achievement in the Cognitive Domains at the Fourth and Eighth Grades

This chapter of the report presents the TIMSS 2003 mathematics achievement results for each of the three cognitive domains. Following the presentation of the results, for each domain in turn - knowing, applying, and reasoning - there is an overview of performance across domains.

## Knowing Facts, Procedures, and Concepts

The first page of Exhibit 2.1 presents the distribution of students' mathematics achievement in the cognitive domain of knowing facts, procedures, and concepts for the 46 countries and four benchmarking entities that participated in TIMSS 2003 at the eighth grade, and the second page presents the distribution of student achievement for the 25 countries and three benchmarking entities that participated at the fourth grade. Countries are shown in decreasing order of average (mean) scale score, together with an indication of whether the country average is significantly higher or lower than the international average. To provide a basis of comparison for the performance of each country in each cognitive domain, the international average across countries for each domain was scaled to be 467 , the same as the international average for mathematics overall. As explained in

Chapter 1 (footnote l), the benchmarking entities were not included in computing the international average. Also, as previously discussed in conjunction with Exhibit 1.1, the years of formal schooling and average age of the students in each country are shown to aid in interpretation of the achievement results. This information also is repeated in Exhibit 2.1 as well as in 2.3 and 2.5 for ease of reference.

Finally, as a reminder that not all countries are equally well equipped to meet the challenge of educating their young people, Exhibit 2.1 as well as Exhibits 2.3 and 2.5 include the value for each country on the Human Development Index provided by the United Nations Development Programme (see Human Development Report 2003). The index has a minimum value of 0 and a maximum of 1.0 . Countries with high values on the index enjoy long life expectancy, high levels of school enrollment and adult literacy, and a good standard of living as measured by per capita GDP. For example, at the eighth grade, TIMSS countries with index values greater than 0.9 included Australia, Belgium (Flemish), England, Israel, Italy, Japan, New Zealand, Norway, The Netherlands, Scotland, Sweden, and the United States. For all three cognitive domains, all of these countries (except Norway in the knowing and applying domains) had average achievement above the international average. However, not all countries performing above the overall international average in the three cognitive domains had an index value as high as 0.9 . Within each of the cognitive domains, the relationship between a country's index value and average student achievement was fairly similar.

As shown in Exhibit 2.1, in the knowing domain for the eighth grade, similar to overall mathematics performance, there was a wide range in performance between the highest- and lowest-performing countries, from 592 in the Republic of Korea to 232 in Ghana. Twentyseven countries and the four benchmarking entities performed above the international average and 17 countries scored below the international average. Moldova and Cyprus performed about the same as the international average.

At the fourth grade, the difference was also large between the highest-performing country Singapore (626) and the lowest-performing country Tunisia (338). Thirteen countries and the three benchmarking entities performed above the international average and eight countries performed below the international average. The four countries performing about at the international average were Australia, Moldova, Cyprus, and New Zealand.

For both the eighth and fourth grades, Exhibit 2.1 illustrates the broad range of achievement both within and across the countries assessed. It provides a graphical representation of student performance within each country. The bar graph for each country shows the 5 th, 25 th, 75 th, and 95 th percentiles ${ }^{1}$ as well as the $95 \%$ confidence for the mean. Each percentile point indicates the percentage of students below that point on the scale. For most TIMSS 2003 participants at the eighth grade, there was an enormous range within each country between the highest and lowest scores, often as much as 400 scale-score points. This range was as large or larger than the difference in mean achievement between the highest and lowest performing country. For the eighth grade knowing scale, the range for most students in the higher-achieving countries was from 400 to 700. In comparison, it tended to be between 300 and 600 for medium-performing countries and from 200 to 500 (or even lower) in the lower-performing countries.

Exhibit 2.2 shows how a country's average mathematics achievement in the knowing domain compares to achievement in the other participating countries. The results for the eighth grade are shown on the first two pages and for the fourth grade on the third page. The figure for each grade shows whether or not the differences in average achievement between pairs of countries are statistically significant. To read the table, select a country of interest from the first column and read across the row corresponding to that country. A circle with a triangle pointing up indicates significantly higher performance than the comparison country listed across the top; absence of a symbol indicates


* Represents years of schooling counting from the first year of ISCED Level 1 .
** Taken from United Nations Development Programme's Human Development Report 2003, p. 237-240.
+ Met guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
$\ddagger$ Nearly satisfied guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
£ Did not satisfy guidelines for sample participation rates (see Exhibit C.2).

1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1)
2 National Defined Population covers less than 90\% of National Desired Population (see Exhibit C.1)

- Korea tested the same cohort of students as other countries, but later in 2003, at the beginning of the next school year.
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.
A dash (-) indicates comparable data are not available.

$\begin{array}{ll}\text { * } \quad \text { Represents years of schooling counting from the first year of ISCED Level } 1 . \\ \text { ** } \quad \text { Taken from United Nations Development Programme's Human Development Report 2003, } \\ \text { p. 237-240. } \\ \dagger \quad & \text { Met guidelines for sample participation rates only after replacement schools were included (see } \\ \text { Exhibit C.2). } \\ 1 & \text { National Desired Population does not cover all of International Desired Population (see Exhibit C.1). }\end{array}$
ø Norway: 4 years of formal schooling, but First Grade is called "First grade/Preschool."
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.
A dash (-) indicates comparable data are not available.

| Exhibit 2.2: | Multiple Comparisons of Average Mathematics Achievement for Knowing <br> Cognitive Domain | мathemarcs <br> Grade |
| :--- | :--- | :--- |

Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries.

| Countries |  | 高 |  |  |  |  |  | $\begin{aligned} & \text { on } \\ & \text { in } \\ & \hline \end{aligned}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Korea, Rep. of |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \cdot$ | 0 | 0 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Singapore |  |  |  |  | 0 | 0 | - | 0 | - 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | $\bigcirc$ | - | 0 | 0 | 0 |  |
| Hong Kong, SAR |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Chinese Taipei |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 00 | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Japan | - | - | - | - |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $0 \cdot$ | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Estonia | - | - | - | - | - |  |  |  | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Belgium (Flemish) | - | - | - | - | - |  |  |  | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | $0 \cdot$ | - 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Hungary | - | - | - | - | - |  |  |  | 0 | - | 0 | 0 | - | 0 | 0 | 0 | 0 | - | 0 - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Netherlands | - | - | - | - | - | - | - | - | - |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Russian Federation | - | - | - | $\bigcirc$ | - | - | - | - |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | $0 \cdot$ | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Latvia | - | - | - | - | - | - | - | - | - |  |  |  |  | 0 | 0 | 0 | 0 | 0 | $0 \cdot$ | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Slovak Republic | - | - | - | - | - | - | - | - | - |  |  |  |  |  | 0 | 0 | 0 | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Lithuania | $\bigcirc$ | - | - | $\bigcirc$ | - | - | - | - | - ${ }^{-}$ |  |  |  |  |  |  |  | 0 | - | $0 \cdot$ | 0 | 0 O | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| United States | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  | 0 | 0 | - | $0 \cdot$ | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Malaysia | - | - | - | - | - | - | - | - | - | - | - | $\bigcirc$ |  |  |  |  |  |  | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Israel | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  | 0 | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Slovenia | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  | 0 | 0 | $0 \cdot$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Australia | - | - | - | - | $\bigcirc$ | - | - | - | - - | - | - | - | - | - |  |  |  |  |  | 0 |  |  |  | 0 | 0 | 0 | 0 | 0 |  |
| Serbia | - | - | - | - | - | - | - | - | - | - | - | $\bigcirc$ | - | - | - |  |  |  |  |  | 0 |  |  | 0 | 0 | 0 | 0 | 0 |  |
| England | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  |  |  |  |  |  | 0 | 0 |  |
| Sweden | - | - | - | - | - | - | - | - | - $\square^{-}$ | - | - | - | - | - | - | - | - | - | $\bigcirc$ |  |  |  |  |  |  |  | 0 | 0 |  |
| Bulgaria | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  | - |  |  |  |  |  |  |  | 0 | 0 |  |
| Romania | $\bigcirc$ | - | - | - | - | - | - | - | - ${ }^{-}$ | $\cdot$ | - | - | - | - | - | - | - |  |  |  |  |  |  |  |  |  | 0 | 0 |  |
| New Zealand | $\bigcirc$ | - | - | - | - | - | - | - | - | - | - | $\bigcirc$ | - | - | - | - | - |  |  |  |  |  |  |  |  |  | 0 | 0 |  |
| Italy | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  |  |  |  | 0 | 0 |  |
| Scotland | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  |  |  |  | 0 | 0 |  |
| Armenia | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  |  |  |  | 0 | 0 |  |
| Moldova, Rep. of | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - ${ }^{-1}$ | - | - | - | - | $\cdots$ | - | - |  |  | 0 |
| Cyprus | - | - | - | $\bigcirc$ | $\bigcirc$ | - | - | - | - | - | - | $\bigcirc$ | - | - | - | - | - | - | - $\cdot$ | - | - ${ }^{-}$ | - | - | - | - | - |  |  | 0 |
| Norway | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - $\cdot$ | - | - | - | - | - | - | - | - | $\cdots$ |  |
| Lebanon | - | - | - | - | - | - | - | - | - $\square^{-1}$ | - | - | - | - | - | - | - | - | - | - $\cdot$ | - | - ${ }^{\circ}$ | - | - | - | - | - | - | - |  |
| Macedonia, Rep. of | - | - | - | - | - | - | - | - | - $\cdot$ | - | - | - | - | - | - | - | - | - | - $\cdot$ | - | - - | - | - | $\bigcirc$ | - | - | - | - |  |
| Jordan | - | - | - | - | - | - | - | - | - $\cdot$ | - | - | $\stackrel{\square}{ }$ | - | - | - | - | ${ }^{\circ}$ | - | (-) | - | $\bigcirc$ | - | - | - | - | - | - | - |  |
| Indonesia | - | - | - | - | - | - | - | - | - - | - | - | - | - | - | - | - | $\bigcirc$ | - | - | - | - | - | - | - | - | - | - | - |  |
| Egypt | - | - | - | - | - | - | - | - | - - | - | - | - | - | - | - | - | - | - | - | - | - - | - | - | - | - | - | - | - |  |
| Iran, Islamic Rep. of | - | - | $\cdots$ | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - ${ }^{-1}$ | - | - - | - | - | - | - | - | - | - |  |
| Bahrain | - | - | - | - | - | - | - | - | - | $\bigcirc$ | - | - | - | - | - | - | - | - | - $\cdot$ | - | - | - | - | - | - | - | - | - |  |
| Tunisia | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - ${ }^{\circ}$ | - | - $\cdot$ | - |  | - | - | - | - | - |  |
| Palestinian Nat'l Auth. | $\bullet$ | ${ }^{\circ}$ | - | - | - | - | - | - | - ${ }^{-}$ | - | - | - | - | ${ }^{\circ}$ | - | - | $\cdot$ | - | - ${ }^{-}$ | - | - ${ }^{-}$ | - | - | ${ }^{\circ}$ | - | - | ${ }^{\circ}$ | ${ }^{\circ}$ |  |
| Philippines | - | - | - | - | - | - | - | - | - | - | - | $\cdots$ | - | - | - | - | - | - | - ${ }^{-1}$ | - | - - | - | - | - | - | - | - | - |  |
| Morocco | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - $\cdot$ | - | - | - | - | - | - | - | - | - |  |
| Chile | - | - | - | - | - | - | - | - | - $\cdot$ | - | - | - | - | - | - | - | - | - | - ${ }^{-1}$ | - | - | - | - | - | - | - | - | - |  |
| Botswana | - | - | - | - | - | - | - | - | - $\cdot$ | - | - | - | - | - | - | - | - | - | - $\cdot$ | - | - | - | - | - | - | - | - | - |  |
| Saudi Arabia | - | $\bigcirc$ | - | - | $\bigcirc$ | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - - | - | - | - | - | - | - | - |  |
| South Africa | - | - | - | $\bigcirc$ | $\bigcirc$ | - | - | - | - ${ }^{-}$ | - | - | - | - | - | - | - | - | - | - | - | - ${ }^{\circ}$ | - | - | - | - | - | - | - |  |
| Ghana | - | - | - | - | - | - | - | - | - ${ }^{-1}$ | - | - | - | - | - | - | - | - | - | - | - | - $\cdot$ | - | - | - | - | - | - | - |  |

Benchmarking Participants

| Basque Country, Spain | - | $\bigcirc$ | - | $\checkmark$ | - | $\bigcirc$ | $\checkmark$ | - | $\bigcirc$ | © | - | - | - | $\bigcirc$ | - |  |  |  |  |  | 0 | 0 |  |  | 0 | 0 | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Indiana State, US | (1) | (1) | - | ( | ( | - | - | - |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Ontario Province, Can. | - | - | - | - | - | - | - | - |  |  |  |  |  |  |  | 0 | - | - | - | 0 | 0 | - | 0 | - | 0 | 0 | - | - | - | 0 |
| Quebec Province, Can. | ( | (1) | - | - | - |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

[^1]Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries.

|  |  | $\begin{aligned} & \text { ᄃ } \\ & \stackrel{\pi}{0} \\ & \text { on } \end{aligned}$ |  | $\begin{aligned} & \stackrel{\rightharpoonup}{2} \\ & \underset{\sim}{3} \end{aligned}$ |  |  | $\frac{\frac{\pi}{n}}{\stackrel{y}{5}}$ |  |  |  | $\stackrel{0}{\bar{ً}}$ | N <br> 0 <br> 0 <br> 0 <br> 0 <br> 0 |  |  |  |  | ~ <br>  |  |  | Countries |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Korea, Rep. of |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Singapore |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Hong Kong, SAR |
| 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Chinese Taipei |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 | Japan |
| 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | Estonia |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 |  | Belgium (Flemish) |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | Hungary |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |  | - | Netherlands |
| 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |  | - | Russian Federation |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |  | (1) | Latvia |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |  | © | Slovak Republic |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |  | (1) | Lithuania |
| 0 | 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | - | - | 0 | - | 0 | 0 |  |  | ( | United States |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |  | - | Malaysia |
| 0 | 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (1) | (1) | (1) | Israel |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | ( | (1) | - | Slovenia |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | © | - | - | Australia |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | © | - | - | Serbia |
| 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 |  | - | - | - | England |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | ( | - | (1) | ( ) | Sweden |
| 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | - | 0 | - | 0 | - | (1) | (1) | - | Bulgaria |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | () | (1) | (1) | Romania |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | - | - | - | - | - | 0 |  | (1) | (1) | - | New Zealand |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | ( | (1) | (1) | - | Italy |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | - | - | 0 | - | 0 | ( | - | - | - | Scotland |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | ( | (1) | - | Armenia |
| 0 | 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | ( | (1) | (1) | ( ) | Moldova, Rep. of |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (-) | - | (1) | - | Cyprus |
|  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | (1) | (1) | - | Norway |
|  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (-) | (1) | (1) | - | Lebanon |
|  |  | $\bigcirc$ | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | - | - | - | - | Macedonia, Rep. of |
| ( ) | - |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | - | - | - | - | Jordan |
| ( ) | - |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | - | - | - | Indonesia |
| ( ) | (-) | - | - |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | - | - | - | - | Egypt |
| (1) | (1) | (1) | ( |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (1) | (1) | (1) | $\bigcirc$ | Iran, Islamic Rep. of |
| ( ${ }^{\text {c }}$ | (-) | - | ( | - |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (-) | - | (1) | - | Bahrain |
| ( | (1) | ( | ( | (1) |  |  |  |  |  | 0 | - | 0 | 0 | - |  | - | - | (1) | - | Tunisia |
| ( ) | - | - | - | © | (-) | - |  |  |  |  |  | 0 | 0 | 0 |  | - | - | - | - | Palestinian Nat'I Auth. |
| ( ${ }^{( }$ | - | - | - | (1) | - | - |  |  |  |  |  | 0 | 0 | 0 |  | - | (1) | - | - | Philippines |
| ( ) | - | - | - | © | (-) | - | © |  |  |  |  | 0 | 0 | 0 |  | - | - | - | - | Morocco |
| (1) | (1) | (1) | (1) | (1) | ( | ( | ( |  |  |  |  | 0 | 0 | 0 | 0 | (1) | (1) | (1) | - | Chile |
| ( ${ }^{\text {c }}$ | (-) | - | (-) | - | (-) | ( ${ }^{\text {c }}$ | ( ) | - | ( | (1) | © |  | 0 | 0 | 0 | (1) | - | (1) | - | Botswana |
| ( | (1) | (1) | (1) | (1) | ( | ( | ( ${ }^{( }$ | - | (1) | ( ${ }^{\text {c }}$ | - | - |  | 0 | 0 | ( | - | - | - | Saudi Arabia |
| ( ) | - | - | - | - | - | - | - | - | - | - ${ }^{(1)}$ | - | - | - |  | 0 | © | - | - | - | South Africa |
| (-) | (-) | - | (1) | (1) | - | (-) | - | - | - | (1) | (-) | (1) | (1) | - |  | (1) | - | (1) | - | Ghana |

- Average achievement significantly higher than comparison country
(v) Average achievement significantly lower than comparison country

| - | 0 | - | - | - | 0 | - | - | - | - | 0 | - | - | - | 0 | 0 |  | (1) | (7) | (7) | Basque Country, Spain |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |  | (7) | Indiana State, US |
| 0 | - | - | - | 0 | 0 | - | 0 | - | - | 0 | - | - | - | - | - | 0 |  |  | (7) | Ontario Province, Can. |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | Quebec Province, Can. |

[^2]
no significant difference in performances; and a circle with a triangle pointing down indicates significantly lower performance.

At the eighth grade, the Republic of Korea, Singapore, Hong Kong SAR, and Chinese Taipei had significantly higher achievement in the knowing domain than the other participating countries. With the exception of those four top-performing countries, Japan had significantly higher achievement than all the rest of the participating countries. Estonia, Belgium (Flemish), and Hungary also performed very well as did the Canadian province of Quebec, being outperformed by only the five top-scoring Asian countries.

At the fourth grade, Singapore had the highest average achievement in the knowing domain followed by Hong Kong SAR and then Chinese Taipei and Japan. Belgium (Flemish) outperformed all the participating countries except the four top-scoring Asian countries. England, the Netherlands, the United States, and the US state of Indiana also had higher average achievement than many of the other participating countries.

## Applying Knowledge and Conceptual Understanding

Exhibit 2.3 presents the distribution of student mathematics achievement in the cognitive domain of applying at the eighth (first page) and fourth (second page) grades.

At the eighth grade, led by Singapore, 24 countries and the four benchmarking participants had achievement in the applying domain significantly higher than the international average. Romania, Bulgaria, Norway, and Serbia performed no differently than the international average and 18 countries performed significantly below this average. At the fourth grade, also led by Singapore, 14 countries and the US state of Indiana had achievement significantly higher than the international average, two countries (Italy and Australia) and the two Canadian provinces had achievement similar to the international average, and 9 countries had achievement below it.


* Represents years of schooling counting from the first year of ISCED Level 1.
** Taken from United Nations Development Programme's Human Development Report 2003, p. 237-240.
$\dagger$ Met guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
$\ddagger$ Nearly satisfied guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2)

Did not satisfy guidelines for sample participation rates (see Exhibit C.2)

1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1).
2 National Defined Population covers less than 90\% of National Desired Population (see Exhibit C.1).

- Korea tested the same cohort of students as other countries, but later in 2003, at the beginning of the next school year.
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.
A dash (-) indicates comparable data are not available.


* Represents years of schooling counting from the first year of ISCED Level 1.
** Taken from United Nations Development Programme's Human Development Report 2003, p. 237-240.
$\dagger$ Met guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1).

Norway: 4 years of formal schooling, but First Grade is called "First grade/Preschool."
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.
A dash $(-)$ indicates comparable data are not available

Exhibit 2.4: \begin{tabular}{ll}

Multiple Comparisons of Average Mathematics Achievement for Applying \& | mathemarics |
| :--- |
| Cognitive Domain | <br>

Grade <br>
\hline
\end{tabular}

Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries.

| Countries |  |  |  |  | $\begin{aligned} & \stackrel{⿺}{0} \\ & \stackrel{0}{0} \\ & \stackrel{\circ}{7} \end{aligned}$ |  |  |  |  |  |  | $\frac{0}{2}$ |  |  |  |  |  |  |  |  |  |  |  | $\begin{aligned} & \stackrel{\pi}{5} \\ & \frac{0}{\omega} \\ & \frac{0}{\omega} \end{aligned}$ | 츤 |  |  |  |  |  |  | $\frac{n}{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Singapore |  | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Hong Kong, SAR | - |  |  |  | 0 |  | 0 | 0 | 0 | - | 0 |  | - 0 | - | - | - | 0 | - | 0 | 0 | - | 0 | - | 0 | 0 | 0 | - | - | 0 | 0 | 0 | 0 |
| Korea, Rep. of | - |  |  |  | 0 |  | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Chinese Taipei | - |  |  |  | 0 |  | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Japan | - | - | - | - |  |  | 0 | 0 | 0 | - | 0 |  | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Netherlands | - | - | - | - | - |  |  |  |  | - | 0 |  | - | - | - | - | 0 | 0 | 0 | 0 | - | 0 | - | - | - | - | 0 | - | 0 | - | 0 | 0 |
| Belgium (Flemish) | - | - | - | - | - |  |  |  |  | 0 | 0 |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Estonia | - | - | - | - | - |  | - |  |  |  | 0 |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Hungary | - | - | - | - | - |  | - | - |  |  | 0 |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Malaysia | - | - | - | - | - |  | - | - | $\bigcirc$ | - |  |  |  |  |  |  |  |  |  |  | - | 0 | - | - | - | 0 | 0 | - | - | 0 | - | 0 |
| Australia | - | - | - | - | - |  | - | - | - | - |  |  |  |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Scotland | - | - | - | - | - |  | - | - | - | - |  |  |  |  |  |  |  |  |  |  |  |  |  | 0 | - | 0 | 0 | 0 | - | 0 | - | 0 |
| Sweden | - | - | - | - | - |  | - | - | - | - |  |  |  |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Latvia | - | - | - | $\cdot$ | $\cdot$ |  | - | - | - | - |  |  |  |  |  |  |  |  |  |  |  |  |  | 0 | - | 0 | 0 | - | 0 | 0 | 0 | 0 |
| England | - | - | - | - | - |  | - | - | - | - |  |  |  |  |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Russian Federation | - | - | - | - | - |  | - | - | - | - |  |  |  |  |  |  |  |  |  |  |  |  |  | 0 | - | 0 | 0 | - | 0 | 0 | 0 |  |
| Slovak Republic | - | - | - | - | - |  | - | - | - | - |  |  |  |  |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| United States | - | - | - | - | - |  | - | - | - | - |  |  |  |  |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Lithuania | - | - | - | - | - |  | - | - | - | - | - |  |  |  |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| New Zealand | - | - | - | - | - |  | - | - | - | - | - |  |  |  |  |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | - | - | - | - | 0 |
| Israel | - | - | - | - | - |  | - | - | - | - | - |  | - |  | - |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Slovenia | - | - | - | - | - |  | - | - | - | - | - |  | - | - | - | - | - | - | - | - | - |  |  |  |  | - | 0 | - | 0 | 0 | 0 | 0 |
| Italy | - | - | - | - | - |  | - | - | - | - | - $\cdot$ |  | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  | 0 | 0 | 0 | 0 | 0 |
| Armenia | - | - | - | - | - |  | - | - | - | - | - |  | - | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  | - | - | - | 0 |
| Romania | - | - | - | - | - |  | - | - | - | - | - |  | - | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  |  |  | 0 | 0 |
| Bulgaria | - | - | - | - | - |  | - | - | - | - | - |  | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  |  | 0 |  |
| Norway | - | - | - | - | - |  | - | - | - | - | - |  | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  | 0 | 0 |
| Serbia | - | - | - | - | - |  | - | - | - | - | - |  | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  |  |  |  | - |
| Moldova, Rep. of | - | - | - | - | - |  | - | - | - | - | - |  | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  |  |
| Cyprus | - | $\bigcirc$ | - | - | - |  | - | - | - | - | - |  | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  |
| Macedonia, Rep. of | $\bigcirc$ | - | - | - | - |  | - | - | - | - | - |  | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Lebanon | - | - | - | - | $\cdots$ |  | - | $\bigcirc$ | $\bullet$ | - | - |  | - | - | - | - | - | - | $\bullet$ | $\bigcirc$ | - | - | - | - | - | - | - | - | - | - | - | - |
| Jordan | - | - | - | - | - |  | - | $\bigcirc$ | - | - | - |  | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Tunisia | - | - | - | - | - |  | - | - | - | - | - |  | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Iran, Islamic Rep. of | $\bigcirc$ | - | - | - | - |  | - | - | - | - | - |  | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Indonesia | - | - | - | - | - |  | - | - | - | - | - |  | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Egypt | - | - | - | - | - |  | - | - | - | - | - |  | - | - | - | - | $\stackrel{\rightharpoonup}{*}$ | - | - | $\bullet$ | - | - | - | - | - | - | - | - | - | - | - | $\bullet$ |
| Bahrain | - | - | - | - | - |  | - | - | - | - | - |  | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Chile | - | - | - | - | - |  | - | - | - | - | - |  | $\bigcirc$ | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Palestinian Nat'l Auth. | $\cdot$ | - | - | - | - |  | - | - | - | - | - |  | - | - | - | - | - | - | $\cdot$ | - | - | - | - | - | - | - | $\cdot$ | - | - | - | - | - |
| Morocco | - | - | - | - | - |  | - | - | - | - | - $\cdot$ |  | - | - | - | - | - | - | - | - | - | - | - | - | $\bullet$ | - | - | - | - | - | - | - |
| Philippines | - | - | - | - | - |  | - | $\bigcirc$ | $\bigcirc$ | - | - |  | - | - | $\bigcirc$ | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Botswana | - | - | - | - | - |  | - | - | - | - | - |  | - | - | - | - | ${ }^{\circ}$ | - | $\stackrel{\square}{\circ}$ | $\bullet$ | - | - | $\bullet$ | $\bigcirc$ | $\bullet$ | $\bullet$ | $\bullet$ | $\cdot$ | $\bigcirc$ | $\cdot$ | - | - |
| Saudi Arabia | - | - | - | - | - |  | - | - | - | - | - |  | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| Ghana | - | - | - | - | - |  | - | (-) | - | - | - |  | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |
| South Africa | - | - | - | - | - |  | - | - | - | - | - |  | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |

## Benchmarking Participants

| Basque Country, Spain | ( | ( | ( | (1) | (1) | - | - | © | (1) | - | - | © | ( | ( | ( | - | ( ) | - | - | ( | - | $\checkmark$ |  |  |  |  | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Indiana State, US | ( | (-) | - | ( | (-) | ( | ( ${ }^{\text {c }}$ | ( ) | (1) |  |  |  |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Ontario Province, Can. | - | (-) | - | (1) | (1) | (-) | (1) |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Quebec Province, Can. | ( | - | - | (1) | ( ) |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\bigcirc$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

[^3]$\left.\begin{array}{|lll}\hline \text { Exhibit 2.4: } & \begin{array}{l}\text { Multiple Comparisons of Average Mathematics Achievement for Applying } \\ \text { Cognitive Domain }\end{array} & \begin{array}{l}\text { mathemarics } \\ \text { Grade }\end{array} \\ 0\end{array}\right)$

Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries.

|  |  |  | $\frac{\frac{\pi}{5}}{5}$ |  |  | $\stackrel{\stackrel{\rightharpoonup}{2}}{\underset{\sim}{3}}$ |  | $\frac{\stackrel{0}{\bar{J}}}{\frac{1}{\bar{J}}}$ |  |  |  |  |  |  |  |  | ~ <br>  |  |  | Countries |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Singapore |
| 0 | - | 0 | 0 | 0 | - | - | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | $\bigcirc$ | - | 0 | 0 | Hong Kong, SAR |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Korea, Rep. of |
| 0 | - | 0 | 0 | 0 | - | - | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | $\bigcirc$ | - | 0 | 0 | Chinese Taipei |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Japan |
| 0 | - | - | - | 0 | 0 | 0 | 0 | - | 0 | 0 | - | - | 0 | 0 | 0 | 0 | 0 | 0 |  | Netherlands |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | Belgium (Flemish) |
| 0 | 0 | - | - | 0 | 0 | 0 | 0 | 0 | - | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 |  | - | Estonia |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | - | Hungary |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (1) | (1) | Malaysia |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (1) | - | Australia |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | © | - | Scotland |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | © | - | Sweden |
| 0 | 0 | - | - | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 |  | - | - | Latvia |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (1) | - | England |
| 0 | - | - | - | 0 | - | - | 0 | - | - | 0 | - | - | - | - | - | 0 |  | - | - | Russian Federation |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (1) | - | Slovak Republic |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | © | - | United States |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (1) | - | Lithuania |
| 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 |  | - | - | New Zealand |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (1) | - | Israel |
| 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | - | 0 | 0 | 0 | (1) | - | (1) | Slovenia |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (-) | (1) | - | Italy |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (1) | - | - | Armenia |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (-) | (1) | - | Romania |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 |  | - | - | - | Bulgaria |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | © | - | (1) | - | Norway |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | - | - | - | Serbia |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | - | (-) | - | Moldova, Rep. of |
| 0 | - | - | 0 | 0 | - | - | - | 0 | - | 0 | 0 | 0 | 0 |  | 0 | - | - | - | - | Cyprus |
|  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | (1) | - | (-) | - | Macedonia, Rep. of |
|  |  |  |  | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 |  | 0 | - | - | - | - | Lebanon |
|  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | - | - | © | - | Jordan |
| (1) |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\checkmark$ | ( | (1) | (1) | Tunisia |
| (1) | - |  |  |  |  | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 |  | 0 | (-) | - | () | - | Iran, Islamic Rep. of |
| (1) | ( | (1) | (1) |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | ( | (1) | - | Indonesia |
| (1) | - | - | - | © |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | (1) | - | (1) | - | Egypt |
| (1) | ( ) | - | (1) | (1) | - |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | - | - | - | Bahrain |
| ( ${ }^{( }$ | - | - | - | © | - | - |  |  |  |  | 0 | 0 | 0 | 0 | 0 | - | - | (1) | - | Chile |
| ( ) | - | - | - | - | - | - | - |  |  |  |  | 0 | 0 | 0 | 0 | - | - | - | - | Palestinian Nat'l Auth. |
| ( ) | - | - | - | © | - | - | - |  |  |  |  | 0 | 0 | 0 | 0 | - | - | (1) | - | Morocco |
| - | - | - | - | - | - | - | (1) | (1) |  |  |  |  | $\bigcirc$ | 0 | 0 | - | - | - | - | Philippines |
| ( | ( | - | - | - | ( | - | - | ( ) | - | (1) |  |  | 0 | 0 | 0 | (1) | - | (1) | - | Botswana |
| ( | ( ${ }^{( }$ | - | (1) | (1) | (-) | - | (1) | (1) | (1) | (1) | (1) | (1) |  | 0 | 0 | (-) | ( | (1) | - | Saudi Arabia |
| ( ${ }^{( }$ | - | - | - | © | - | ( ) | - | - | - | (*) | - | (1) | - |  | 0 | © | - | (-) | - | Ghana |
| ( | - | - | - | - | - | - | - | - | ( | - | - | (1) | - | - |  | - | ( | (1) | - | South Africa |

- Average achievement significantly higher than comparison country
(v) Average achievement significantly lower than comparison country

| - | 0 | - | - | 0 | 0 | - | - | - | - | 0 | - | - | - | 0 | 0 |  | (1) | () | () | Basque Country, Spain |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (7) | (7) | Indiana State, US |
| 0 | - | - | - | 0 | 0 | - | 0 | - | 0 | 0 | - | - | 0 | - | 0 | 0 | - |  | (7) | Ontario Province, Can. |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | Quebec Province, Can. |

[^4]Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries.


Benchmarking Participants

| Indiana State, US | $\bigcirc$ | - | - | - | ( | (1) | (1) | - | - | ) |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ontario Province, Can. | - | - | - | - | - | (1) | - | - | v | - | $\bigcirc$ | (-) | - |  |  |  |  |  | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | - |  |
| Quebec Province, Can. | - | - | - | - | - | $\bigcirc$ | - | $\checkmark$ | - | ) | - | - | - |  |  |  |  | $\bigcirc$ | 0 | - | - | 0 | - | - | - | 0 | 0 | - |  |

- Average achievement significantly higher than comparison country
- Average achievement significantly lower than comparison country

Exhibit 2.4 shows for the eighth (first two pages) and fourth (third page) grades how a country's average mathematics achievement in the applying domain compares to achievement in the other participating countries.

At the eighth grade, Singapore had the highest achievement in the applying domain. Hong Kong SAR, the Republic of Korea, and Chinese Taipei all performed equally well, but not as well as Singapore. These countries were followed by Japan with performance below that of the four top-scoring countries, but with significantly higher achievement in this domain than all of the other participants at the eighth grade. The Netherlands and Belgium (Flemish) were outperformed by the five top-scoring Asian countries, but also did very well.

At the fourth grade, results for the four Asian countries in the applying domain were nearly the same as for the knowing domain. The Singaporean students had the highest average achievement in the applying domain, followed by the students in Hong Kong SAR, who (with the exception of Singapore) had significantly higher achievement than students in the other participating countries. Japan and Chinese Taipei performed similarly to each other and had the next highest achievement after Hong Kong SAR. Compared to the knowing domain, however, several more countries performed similarly to each other and were in the second highest achieving group of countries. Belgium (Flemish), Latvia, the Russian Federation, Lithuania, and the Netherlands had significantly higher achievement in the applying domain than the rest of the participating countries and benchmarking entities.

## Reasoning

The first and second pages of Exhibit 2.5 show the distribution of student mathematics achievement in the cognitive domain of reasoning at the eighth and fourth grades, respectively. Exhibit 2.6 shows, for the eighth and fourth grades (first two pages and third page, respectively), how a


* Represents years of schooling counting from the first year of ISCED Level 1 .
** Taken from United Nations Development Programme's Human Development Report 2003, p. 237-240.

Met guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
$\ddagger$ Nearly satisfied guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
\# Did not satisfy guidelines for sample participation rates (see Exhibit C.2).
National Desired Population does not cover all of International Desired Population (see Exhibit C.1).



* Represents years of schooling counting from the first year of ISCED Level 1.
** Taken from United Nations Development Programme's Human Development Report 2003, p. 237-240.
$\dagger$ Met guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1).
ø Norway: 4 years of formal schooling, but First Grade is called "First grade/Preschool."
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.
A dash (-) indicates comparable data are not available.

Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries.


## Kore

Singapore





Instructions: Read across the row for a country to compare performance with the countries listed along the top of the chart. The symbols indicate whether the average achievement of the country in the row is significantly lower than that of the comparison country, significantly higher than that of the comparison country, or if there is no statistically significant difference between the average achievement of the two countries.

|  |  | $\frac{. \frac{\mathrm{C}}{0}}{\frac{\frac{0}{N}}{\frac{1}{0}}}$ |  | $$ | $$ |  | Palestinian Nat'I Auth. |  | $\begin{aligned} & \frac{0}{n} \\ & \frac{0}{\frac{2}{2}} \\ & \end{aligned}$ |  |  | 0 0 0 0 0 0 0 | $\begin{aligned} & \frac{0}{0} \\ & \frac{0}{0} \\ & \frac{0}{4} \\ & \frac{0}{0} \\ & \frac{0}{0} \\ & n \end{aligned}$ |  |  | Basque Country, Spain | n <br>  |  |  | Countries |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 |  | 0 | 0 | 0 | 0 | 0 | Singapore |
| 0 | 0 | - | - | 0 | - | - | - | - | - | - | - | - | 0 |  | 0 | 0 | - | - | 0 | Korea, Rep. of |
| 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Chinese Taipei |
| - | - | 0 | - | - | - | - | - | - | - | - | - | - | 0 |  | - | 0 | - | - | - | Japan |
| 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 | 0 | Hong Kong, SAR |
| - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  | Netherlands |
| 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 |  |  | Belgium (Flemish) |
| - | - | - | - | 0 | - | - | - | - | - | - | - | - | - | - | - | 0 | - |  | (-) | Hungary |
| 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 |  | (7) | Estonia |
| - | - | 0 | - | - | - | - | - | - | - | 0 | - | - | 0 |  | - | 0 |  | ( $\downarrow$ | (-) | Australia |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 |  | (-) | (-) | Scotland |
| - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 0 |  | - | - | New Zealand |
| 0 | - | 0 | 0 | 0 | 0 | - | - | 0 | 0 | 0 | - | 0 | - |  | 0 | 0 |  | ( ) | (1) | England |
| 0 | - | - | - | - | 0 | 0 | - | - | - | 0 | - | - | - |  | - | 0 |  | (7) | (1) | Sweden |
| 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (-) | (7) | United States |
| 0 | 0 | 0 | - | - | - | - | - | - | - | 0 | 0 | 0 | - | - | - | 0 |  | (-) | (-) | Slovak Republic |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | (v) | (-) | Malaysia |
| - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |  |  | (-) | (-) | Latvia |
| 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 |  |  | ( | (-) | Russian Federation |
| - | - | - | - | 0 | - | - | 0 | - | - | - | - | - | - | - | - |  |  | (1) | (-) | Slovenia |
| 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | ( $)$ | (7) | (7) | Lithuania |
| - | - | 0 | - | - | - | - | - | - | - | 0 | - | - | 0 | - | - |  | (1) | ( $\downarrow$ | (-) | Italy |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | (1) | - | (-) | (-) | Israel |
| - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | (1) | ( ) | (7) | (-) | Norway |
| 0 | - | 0 | 0 | 0 | 0 | - | - | 0 | 0 | 0 | - | 0 | - | 0 | 0 | (7) | ( ) | ( ) | (1) | Bulgaria |
| 0 | - | - | - | 0 | 0 | 0 | - | - | - | 0 | - | - | - | - | - | (1) | (1) | () | (1) | Armenia |
| 0 | - | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | (-) | (-) | (-) | (7) | Serbia |
| - | - | 0 | - | - | - | - | - | - | - | 0 | - | - | 0 | - | - | ( ) | ( ) | - | (-) | Romania |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | (v) | (-) | (1) | (-) | Cyprus |
| - | 0 | - | - | 0 | - | - | 0 | - | - | - | 0 | - | - | - | 0 | ( ) | (1) | (-) | (-) | Moldova, Rep. of |
|  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | (-) | (i) | ( - | (-) | Macedonia, Rep. of |
|  |  | - | - | 0 | 0 | - | - | - | - | - | - | - | - | - | - | (7) | ( ) | (-) | (-) | Jordan |
| (7) | (7) |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | (-) | (-) | (-) | (1) | Bahrain |
| (7) | (1) | (-) |  |  |  | - | - | 0 | - | 0 | 0 | 0 | - | - | - | (-) | ( ) | (7) | (-) | Iran, Islamic Rep. of |
| (7) | (1) | (-) |  |  |  |  |  | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | (7) | (-) | (-) | (7) | Lebanon |
| (7) | (1) | (7) |  |  |  |  |  |  | - | 0 | - | - | - | - | - | () | ( ) | (7) | (-) | Chile |
| (7) | (1) | (1) | (1) |  |  |  |  |  |  | 0 | 0 | 0 | - | 0 | 0 | (7) | ( ) | (1) | (1) | Indonesia |
| (7) | (7) | (1) | (1) |  |  |  |  |  |  | 0 | - | - | - | - | - | (-) | () | (7) | (-) | Palestinian Nat'I Auth. |
| (7) | (1) | (1) | (1) | (1) |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | () | ( $)$ | (7) | (7) | Egypt |
| (7) | (1) | (7) | (7) | (1) | ( ) |  |  |  |  |  | - | - | - |  | - | (7) | (7) | (7) | (7) | Tunisia |
| (-) | (1) | (-) | (7) | (-) | (-) | (1) | (7) |  |  |  | 0 | 0 | 0 | 0 | 0 | (v) | (-) | (1) | (-) | Morocco |
| (7) | (1) | (1) | (1) | (1) | - | (-) | (1) | (-) | (1) | ( ) |  |  |  |  | 0 | $\checkmark$ | (1) | ( ) | (-) | Philippines |
| (-) | - | (-) | - | (-) | - | (-) | (-) | (v) | (1) | (\%) |  |  |  |  | 0 | (v) | - | - | (-) | Botswana |
| $\checkmark$ | () | (-) | (-) | (-) | - | (-) | - | ( ) | (-) | - |  |  |  |  | - | - | - | - | (-) | Saudi Arabia |
| (7) | (1) | (7) | (1) | (7) | (1) | (1) | (1) | (7) | (1) | (7) | (7) | (1) | (7) |  | 0 | (7) | () | (1) | (1) | Ghana |
| (7) | (1) | (1) | (7) | (7) | (1) | (7) | (1) | (1) | (7) | (7) | (7) | (7) | (7) | (7) |  | (7) | (1) | (7) | (7) | South Africa |

Benchmarking Participants

| 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | - | 0 | 0 | 0 | 0 | - | - | 0 | 0 | 0 |  |  | (1) | (1) | Basque Country, Spain |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | - | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 | 0 | 0 | 0 |  |  | $\checkmark$ | - | Indiana State, US |
| 0 | - | - | - | - | 0 | - | - | - | - | - | - | 0 | - | - | - | - | 0 | 0 | 0 | 0 |  | $\checkmark$ | Ontario Province, Can. |
| 0 | 0 | - | 0 | 0 | 0 | - | - | - | - | 0 | 0 | 0 | - | - | - | - | 0 | 0 | 0 | 0 | - |  | Quebec Province, Can. |

- Average achievement significantly higher than comparison country
(7) Average achievement significantly lower than comparison country

[^5]

- Average achievement significantly higher than comparison country
(7) Average achievement significantly lower than comparison country
country's average mathematics achievement in the reasoning domain compares to achievement in the other participating countries.

At the eighth grade, average achievement in the reasoning domain ranged from 583 in Singapore to 287 in South Africa. Twentyfour countries and the four benchmarking participants performed significantly above the international average, three countries (Bulgaria, Armenia, and Serbia) performed comparably to the international average, and 19 countries performed significantly below the average.

At the eighth grade, looking at both Exhibits 2.5 and 2.6, it can be seen that the rank ordering of significant differences in achievement is rather complicated for the reasoning domain. Singapore and the Republic of Korea had the highest average achievement in the reasoning domain, nearly identical ( 583 and 582), but Singapore had a larger standard error ( 3.5 to 1.7 ). Thus, the Republic of Korea had significantly higher achievement than every participating country except Singapore and Chinese Taipei whereas Singapore (with the larger standard error) had higher average achievement than every participating country except the Republic of Korea, Chinese Taipei, and Japan. Chinese Taipei and Japan had the same average score (576) followed by Hong Kong SAR. Chinese Taipei (also with a relatively larger standard error of 4.2) did not perform statistically differently than the other three Asian countries, whereas a difference was found between the Republic of Korea and Japan due to their small standard errors. Hong Kong SAR was outperformed only by Singapore and the Republic of Korea. The Netherlands and Belgium (Flemish) only were outperformed by the five top-scoring Asian countries.

At the fourth grade, performance ranged from 574 for Singapore to 340 for Tunisia. Fifteen countries and the three benchmarking participants performed significantly above the international average, three countries (Italy, Scotland, and Moldova) performed essentially at the international average, and seven countries performed significantly below the international average. Singapore had the highest achievement, outperforming all countries except Hong Kong SAR and

Chinese Taipei. Hong Kong, Chinese Taipei and Japan had similar achievement followed by Belgium (Flemish), England, and the Netherlands (all with similar average achievement and only outperformed by the four highest-achieving Asian countries).

## Overview Across Domains

At both the eighth and fourth grades, the countries with the highest achievement in each of the three cognitive domains also tended to be the highest-scoring countries (though not always in the same rank order) on the overall mathematics assessment. At the eighth grade (see Exhibit 1.1), the four countries with the highest overall mathematics achievement were Singapore followed by the Republic of Korea, Hong Kong SAR, and Chinese Taipei (only outperformed by Singapore). Japan had the next highest achievement outperforming all the rest of the participating countries except the previous four countries. Belgium (Flemish), the Netherlands, Estonia, Hungary, and the Canadian province of Quebec also performed well (at least as well or better than all other participants except the five Asian countries listed above).

- In knowing, similar to overall mathematics achievement, the Republic of Korea, Singapore, Hong Kong SAR, and Chinese Taipei had the highest achievement followed by Japan (see Exhibits 2.1 and 2.2). As a slight difference compared to the results for overall mathematics achievement, the four top-scoring Asian countries performed similarly to each other in the knowing domain. Estonia, Belgium (Flemish), Hungary, and the Canadian province of Quebec were outperformed only by the five top-achieving Asian countries.
- In applying, Singapore had the highest average achievement followed by Hong Kong SAR, the Republic of Korea, and Chinese Taipei and then Japan (see Exhibits 2.3 and 2.4). The Netherlands, Belgium (Flemish), and the Canadian province of Quebec were next (only outperformed by the five top-achieving Asian countries).
- In reasoning, Singapore and the Republic of Korea performed very similarly followed by Chinese Taipei and Japan and then Hong Kong SAR (see Exhibits 2.5 and 2.6). The Netherlands, Belgium (Flemish), and the Canadian province of Quebec also had relatively high achievement, only being outperformed by the five Asian countries.

At the fourth grade, Singapore was the highest-performing country in overall mathematics followed by Hong Kong SAR, and then by Japan and Chinese Taipei who performed similarly (see Exhibit 1.1). Belgium (Flemish) had higher achievement than all countries except these four Asian countries.

- In knowing, the pattern at the fourth grade was the same as for overall mathematics (see Exhibits 2.1 and 2.2). The four Asian countries had the best achievement (Singapore followed by Hong Kong SAR, and then by Chinese Taipei and Japan) with Belgium (Flemish) having higher achievement than all countries except the four bestachieving Asian countries.
- In applying, the pattern for the four high-achieving Asian countries was the same as for overall mathematics (see Exhibits 2.3 and 2.4). However, Belgium (Flemish), Latvia, the Russian Federation, Lithuania, and the Netherlands all followed, performing similarly to each other with lower achievement than the four Asian countries, but higher achievement than the rest of the participating countries.
- In reasoning, Singapore, Hong Kong SAR, and Chinese Taipei had the highest achievement (see Exhibits 2.5 and 2.6). Japan had achievement similar to Hong Kong SAR and Chinese Taipei, but was outperformed by Singapore. Belgium (Flemish), England, and the Netherlands had achievement equal to or higher than all participants except the four top-achieving Asian countries.

Just as countries with high achievement on the mathematics assessment as a whole had high achievement in the three cognitive domains, countries scoring lowest on the assessment as a whole
(e.g., South Africa, Ghana, and Saudi Arabia at the eighth grade and the Philippines, Morocco, and Tunisia at the fourth grade) also tended to have low performance in all three cognitive domains. For some middle-performing countries, however, performance was more varied. For example, at the eighth grade, Armenia performed above the international average in the knowing and applying domains (480 and 478) but essentially at the international average in the reasoning domain (468).

Looking at the range in scale scores across the cognitive domains at the eighth grade, the differences in average achievement between the highest- and lowest-performing countries were largest in the knowing domain ( 360 score points), next largest in the applying domain (342), and smallest in the reasoning domain (296). As described in the following sections, several more countries performed significantly above the international average in the knowing domain than in the applying and reasoning domains.

At the fourth grade, with fewer countries, the range in performance between the highest- and lowest-performing countries was smaller than at the eighth grade, but the pattern was similar. The largest difference was in the knowing domain (288), next in the applying domain (246), and the smallest difference was in the reasoning domain (234). In each of the three cognitive domains, about the same number of participants performed above, similar to, or below the international average.


## Chapter 3

## Achievement by Gender in the Mathematics Cognitive Domains at the Fourth and Eighth Grades

This chapter presents average achievement by gender for the three mathematics cognitive domains. In general, as described in Chapter 1 in conjunction with Exhibit 1.1, on average, across the TIMSS 2003 participating countries and benchmarking entities, there was essentially no difference in achievement between boys and girls at either the eighth or fourth grade. Within the cognitive domains, however, there were significant differences by gender, especially at the eighth grade.

At the eighth grade, girls had the advantage in more countries in the knowing domain of mathematics and, even more so in the reasoning domain. Internationally across the TIMSS 2003 participants, girls had significantly higher achievement, on average, than boys in both these domains. Boys had the advantage in more countries in the applying domain.

At the fourth grade, while performance was about the same internationally for boys and girls in the knowing domain, there was a significant difference, on average, favoring boys in the applying domain. Also, boys had significantly higher achievement in considerably more countries than did girls. In the reasoning domain, there was
essentially no difference internationally between boys and girls, but in the few countries where significant differences were found, girls had higher performance.

## Gender Differences in the Knowing Cognitive Domain

Exhibit 3.1 shows gender differences in eighth-grade (first page) and fourth-grade (second page) mathematics achievement in the knowing domain. For each grade and for each country, it presents average achievement separately for girls and boys for each of the TIMSS 2003 participants, as well as the difference between the means. Countries are shown in increasing order of this gender difference. The gender difference for each country is shown by a bar indicating the amount of the difference, whether the direction of the difference favored girls or boys, and whether the difference is statistically significant (indicated by a darkened bar).

At the eighth grade, there was a small but significant difference favoring girls, on average, across countries in the knowing domain. Girls had significantly higher achievement than boys in the knowing domain in nearly half the countries ( 18 countries and the Basque Country, Spain). In contrast, boys had significantly higher achievement than girls in four countries.

At the fourth grade, there essentially was no difference internationally in achievement in the knowing domain between boys and girls. There were differences in some countries, with girls outperforming boys in about the same number of countries as boys outperformed girls. Girls had significantly higher achievement than boys in four countries while boys had significantly higher achievement than girls in four countries and the two Canadian provinces.

$\dagger$ Met guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
$\ddagger$ Nearly satisfied guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
\# Did not satisfy guidelines for sample participation rates (see Exhibit C.2).

1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1).
2 National Defined Population covers less than $90 \%$ of National Desired Population (see Exhibit C.1).

- Korea tested the same cohort of students as other countries, but later in 2003, at the beginning of the next school year.
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Exhibit 3.1: Average Mathematics Achievement by Gender for Knowing Cognitive Domain


1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1)
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

## Gender Differences in the Applying Cognitive Domain

Exhibit 3.2 shows achievement differences between girls and boys for the applying domain for the eighth and fourth grades, (on the first and second pages, respectively). For the applying domain at the eighth grade, boys had significantly higher achievement in more countries than girls. Girls had significantly higher achievement than boys in seven countries, and boys had significantly higher achievement than girls in 13 countries and two benchmarking participants (the US state of Indiana and the Canadian province of Quebec).

Fourth grade had a corresponding pattern for the applying domain, with boys having significantly higher achievement in more countries than girls. Girls had higher achievement in the applying domain in four countries whereas boys had higher achievement in seven countries and the two Canadian provinces. Also, internationally, on average, there was a small but significant difference favoring boys.

$\dagger$ Met guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
$\ddagger \quad$ Nearly satisfied guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
$\ddagger$ Did not satisfy guidelines for sample participation rates (see Exhibit C.2).

1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1).
2 National Defined Population covers less than $90 \%$ of National Desired Population (see Exhibit C.1).

- Korea tested the same cohort of students as other countries, but later in 2003, at the beginning of the next school year.
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.


## Exhibit 3.2: Average Mathematics Achievement by Gender for Applying Cognitive Domain


$\square$ Gender difference not statistically significant

1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1).
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

## Gender Differences in the Reasoning Cognitive Domain

Exhibit 3.3 shows gender achievement differences in the reasoning domain at the eighth grade (first page) and fourth grade (second page). On average, across all countries, eighth-grade girls had significantly higher achievement than boys in the reasoning domain. In this domain, girls had significantly higher achievement than boys in 17 countries and the Basque Country, Spain whereas boys had higher achievement in only two countries (Morocco and Tunisia).

At the fourth grade this pattern was similar, but far less pronounced. There was essentially no difference in achievement internationally between fourth-grade boys and girls in the reasoning domain. However, girls had higher achievement than boys in three countries whereas boys did not outperform girls in any country or benchmarking entity.

## Exhibit 3.3: Average Mathematics Achievement by Gender for Reasoning Cognitive Domain



- Met guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
$\ddagger \quad$ Nearly satisfied guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
\# Did not satisfy guidelines for sample participation rates (see Exhibit C.2).

1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1).
2 National Defined Population covers less than $90 \%$ of National Desired Population (see Exhibit C.1).

- Korea tested the same cohort of students as other countries, but later in 2003, at the beginning of the next school year.
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.


## Exhibit 3.3: Average Mathematics Achievement by Gender for Reasoning Cognitive Domain

| Countries | Girls |  | Boys |  | Difference <br> (Absolute Value) | Gender Difference |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Percent of Students | Average Scale Score | Percent of Students | Average Scale Score |  | Girls <br> Scored Higher |  | Boys <br> Scored Higher |
| Latvia | 49 (0.9) | 531 (3.3) | 51 (0.9) | 531 (4.1) | 0 (3.9) |  |  |  |
| Belgium (Flemish) | 50 (1.0) | 541 (2.6) | 50 (1.0) | 541 (2.8) | 0 (3.1) |  |  |  |
| ${ }^{\dagger}$ Australia | 50 (1.0) | 507 (3.9) | 50 (1.0) | 507 (4.2) | 0 (3.4) |  |  |  |
| International Avg. | 49 (0.2) | 496 (0.9) | 51 (0.2) | 495 (0.8) | 1 (0.9) |  |  |  |
| Slovenia | 48 (1.1) | 486 (3.0) | 52 (1.1) | 485 (3.6) | 1 (4.2) |  |  |  |
| Hungary | 50 (0.9) | 525 (4.0) | 50 (0.9) | 524 (3.8) | 1 (4.4) |  |  |  |
| Tunisia | 48 (0.9) | 340 (5.8) | 52 (0.9) | 339 (4.7) | 1 (6.3) |  |  |  |
| ${ }^{\dagger}$ Hong Kong, SAR | 47 (1.1) | 565 (4.0) | 53 (1.1) | 563 (3.8) | 2 (2.7) |  |  |  |
| New Zealand | 50 (1.1) | 502 (2.9) | 50 (1.1) | 504 (2.4) | 2 (3.1) |  |  |  |
| Cyprus | 49 (0.7) | 515 (2.7) | 51 (0.7) | 517 (3.0) | 2 (3.1) |  |  |  |
| ${ }^{1}$ Lithuania | 49 (0.9) | 527 (3.7) | 51 (0.9) | 529 (3.9) | 2 (3.6) |  |  |  |
| $\dagger$ England | 50 (0.9) | 539 (4.0) | 50 (0.9) | 536 (4.2) | 3 (4.2) |  |  |  |
| Chinese Taipei | 48 (0.5) | 565 (2.6) | 52 (0.5) | 562 (2.7) | 3 (2.9) |  |  |  |
| ${ }^{+}$Netherlands | 49 (1.1) | 533 (3.4) | 51 (1.1) | 536 (3.2) | 4 (3.2) |  | - |  |
| Morocco | 49 (1.1) | 366 (5.6) | 51 (1.1) | 370 (4.7) | 4 (5.4) |  | 1 |  |
| Norway | 50 (0.8) | 466 (2.5) | 50 (0.8) | 470 (2.8) | 4 (3.3) |  | - |  |
| Russian Federation | 50 (0.7) | 524 (5.2) | 50 (0.7) | 528 (4.9) | 4 (3.4) |  | - |  |
| $\dagger$ United States | 50 (0.5) | 517 (2.6) | 50 (0.5) | 522 (2.9) | 5 (2.5) |  |  |  |
| Japan | 49 (0.6) | 559 (2.1) | 51 (0.6) | 564 (2.6) | 6 (3.2) |  |  |  |
| Italy | 48 (0.8) | 496 (4.7) | 52 (0.8) | 502 (4.1) | 6 (3.7) |  |  |  |
| Armenia | 49 (0.8) | 449 (3.4) | 51 (0.8) | 442 (3.4) | 7 (2.9) |  |  |  |
| + Scotland | 51 (1.0) | 495 (3.5) | 49 (1.0) | 502 (4.0) | 7 (4.3) |  |  |  |
| Singapore | 49 (1.4) | 578 (6.2) | 51 (1.4) | 570 (6.8) | 8 (4.7) |  |  |  |
| Iran, Islamic Rep. of | 39 (4.2) | 406 (6.0) | 61 (4.2) | 396 (4.3) | 10 (7.7) |  |  |  |
| Moldova, Rep. of | 50 (0.8) | 501 (5.5) | 50 (0.8) | 488 (5.6) | 13 (5.2) |  |  |  |
| Philippines | 51 (1.0) | 366 (8.8) | 49 (1.0) | 352 (6.6) | 13 (5.7) |  |  |  |
| Benchmarking Participants |  |  |  |  |  |  |  |  |
| Indiana State, US | 52 (1.1) | 528 (3.9) | 48 (1.1) | 528 (4.1) | 0 (4.3) |  |  |  |
| Ontario Province, Can. | 48 (1.1) | 520 (3.4) | 52 (1.1) | 525 (4.8) | 6 (4.1) |  |  |  |
| Quebec Province, Can. | 50 (0.9) | 510 (3.0) | 50 (0.9) | 514 (3.2) | 4 (3.6) |  | - |  |
|  |  |  |  |  |  | Gender difference statistically significant <br> $\square$ Gender difference not statistically significant |  |  |

National Desired Population does not cover all of International Desired Population (see Exhibit C.1)
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.


## Chapter 4 Country by Country Profiles of Achievement in the Mathematics Cognitive Domains

To highlight relative strengths and weaknesses within each country, this chapter describes in which mathematics cognitive areas each country is relatively strong or weak. Regardless of international standing, the profiles of achievement within country reveal that many countries performed relatively better or worse in one or more cognitive domains than they did overall.

Differences in relative performance may be related to one or more of a number of factors, such as emphases in intended curriculum or widely used textbooks, differences in instruction and curriculum implementation, and differences in the match between instruction and the types of items contained in TIMSS 2003.

## Profiles of Achievement

For each country, Exhibit 4.1 displays the difference between average performance in each content area and the country's average performance overall. The first three pages of Exhibit 4.1 show the results for eighth grade and the next two pages show the results for the
fourth grade. For each country, the average of the cognitive domain scores has been set to zero, so that above average or below average performance can be highlighted for each of the three domains. Relatively better achievement in a cognitive domain is shown when the circle and the lines indicating its confidence interval are completely above and not touching zero on the scale, and relatively worse achievement by a circle and its confidence interval lines completely below "0."

The profiles of relative performance reveal interesting differences among countries. Most countries show the profile of performing relatively better or worse in only one of the domains, or perhaps having a relative strength in one domain together with a relative weakness in another of the domains. However, a few countries were very balanced in their performance across the cognitive domains, for example, Belgium (Flemish) at the eighth grade and Chinese Taipei at the fourth grade. At the other end of the continuum, a few countries had a relative strength or weakness in each of the three domains. For example, at the eighth grade, it can be seen that Bahrain performed relatively better in the reasoning domain and relatively worse in the knowing and applying domains compared to its average achievement overall. At the fourth grade, the only country with this pattern was Norway, with relatively worse performance in the knowing and applying domains, combined with better performance in the reasoning domain.

## Relative Strengths and Weaknesses in the Knowing Domain

At the eighth grade, countries with relative strength in the knowing domain included Botswana, Bulgaria, Cyprus, Estonia, Hong Kong SAR, Israel, the Republic of Korea, Latvia, Lebanon, Lithuania, the Philippines, Romania, the Russian Federation, Serbia, and the Slovak Republic. The countries that performed significantly less well in the knowing domain than in mathematics overall included Australia, Bahrain, Chile, Ghana, Iran, the Netherlands, New Zealand, Norway, Saudi Arabia, Scotland, Sweden, England, and the Canadian province of Ontario. At the

## Exhibit 4.1: Profiles of Within-Country Relative Performance in Mathematics Cognitive Domains



[^6]- Korea tested the same cohort of students as other countries, but later in 2003, at the beginning of the next school year


## Exhibit 4.1: Profiles of Within-Country Relative Performance in Mathematics матниматітs $(0)$ Cognitive Domains



[^7]Exhibit 4.1: Profiles of Within-Country Relative Performance in Mathematics Cognitive Domains


[^8]
## Exhibit 4.1: Profiles of Within-Country Relative Performance in Mathematics Cognitive Domains <br> Grade



[^9]Exhibit 4.1: Profiles of Within-Country Relative Performance in Mathematics

fourth grade, countries with a relative strength in the knowing domain were Belgium (Flemish), Italy, Singapore, the United States, and the US state of Indiana. Comparatively more countries at the fourth grade had a relative weakness in the knowing domain, including Cyprus, Latvia, Lithuania, the Netherlands, Norway, the Russian Federation, and Slovenia.

## Relative Strengths and Weaknesses in the Applying Domain

At the eighth grade, there were fewer countries with differences between overall mathematics achievement and achievement in the applying domain than there were with such differences in the knowing domain. Countries with a relative strength in the applying domain at the eighth grade included Ghana, Singapore, and Tunisia. Those with a relative weakness in the applying domain included Bahrain, Macedonia, and Serbia.

At the fourth grade, Armenia, Latvia, Lithuania, and the Russian Federation had applying as a particular strength. Compared to performance in overall mathematics, applying was a relative weakness in Australia, New Zealand, Norway, the United States, the US state of Indiana, and the two Canadian provinces (a group including three English-speaking countries).

## Relative Strengths and Weaknesses in the Reasoning Domain

Countries with the reasoning domain as a particular strength at the eighth grade included Bahrain, Chile, Ghana, Japan, Norway, the Palestinian National Authority, Saudi Arabia, Scotland, South Africa, and Sweden. Countries that performed less well in the reasoning domain than they did in overall mathematics included Armenia, Botswana, Cyprus, Hong Kong, Israel, Lebanon, Lithuania, the Philippines, Romania, the Russian Federation, Serbia, and Singapore.

At the fourth grade, the participants with a relative strength in reasoning were Cyprus, New Zealand, Norway, Scotland, Slovenia, and
the two Canadian provinces. Only two countries, Belgium (Flemish) and Singapore, did relatively less well in reasoning as compared to their overall mathematics performance.

## International Achievement Across the Cognitive Domains

At the eighth grade across the TIMSS 2003 participants, the knowing domain had the most differences, with many countries showing either a relative strength or weakness in this area. Fifteen countries performed better in the knowing domain than they did in mathematics overall, and 12 countries and the Canadian province of Ontario performed worse. The applying domain was the cognitive area least likely to feature either relatively strong or relatively weak performance. Only three countries performed better in the applying domain than they did in mathematics overall (Ghana, Singapore, and Tunisia) and only three countries performed worse (Bahrain, Macedonia, and Serbia).

In the reasoning domain at the eighth grade, 10 countries performed relatively better than they did in mathematics overall and 12 countries did less well. The countries making up each of the two groups included those from very different parts of the world geographically and with disparate cultures and mathematics traditions. For example, the countries with a relative strength in the reasoning domain were Bahrain, Chile, Ghana, Japan, Norway, the Palestinian National Authority, Saudi Arabia, Scotland, South Africa, and Sweden.

At the fourth grade, looking across the participating countries, about the same number of differences (strengths or weaknesses) occurred in each of the cognitive domains. However, several more countries showed a relative weakness in the knowing cognitive domain (seven) than had this domain as a relative strength (five). Similarly, more countries had a relative weakness in the applying domain (seven) than had this domain as a relative strength (four). In comparison, more countries showed a relative strength in the reasoning domain (seven) than showed this domain as a relative weakness (two).

## References

Gonzalez, E.J., Galia, J., \& Li, I. (2004). Scaling methods and procedures for the TIMSS 2003 mathematics and science scales. In M.O. Martin, I.V.S. Mullis, \& S.J. Chrostowski (Eds.), TIMSS 2003 technical report. Chestnut Hill, MA: TIMSS \& PIRLS International Study Center, Lynch School of Education, Boston College.

Martin, M.O., Mullis, I.V.S., \& Chrostowski, S.J. (2004), TIMSS 2003 technical report. Chestnut Hill, MA: TIMSS \& PIRLS International Study Center, Lynch School of Education, Boston College.

Mullis, I.V.S., Martin, M.O., Smith, T.A., Garden, R.A., Gregory, K.D., Gonzalez, E.J., Chrostowski, S.J., \& O'Connor, K.M. (2003), TIMSS assessment frameworks and specifications 2003 (2nd Edition). Chestnut Hill, MA: TIMSS \& PIRLS International Study Center, Lynch School of Education, Boston College.

Mullis, I.V.S., Martin, M.O., Gonzalez, E.J., \& Chrostowski, S.J. (2004), TIMSS 2003 international mathematics report. Chestnut Hill, MA: TIMSS \& PIRLS International Study Center, Lynch School of Education, Boston College.

Mullis, I.V.S., Martin, M.O., \& Gonzalez, E.J. (2004), International achievement in the processes of reading achievement: results from PIRLS 2001 in 35 countries. Chestnut Hill, MA: TIMSS \& PIRLS International Study Center, Lynch School of Education, Boston College.

Mullis, I.V.S., Martin, M.O., Ruddock, G.J., O'Sullivan, C.Y., Arora, A., \& Erberber, E. (2005), TIMSS 2007 assessment frameworks, Chestnut Hill, MA: TIMSS \& PIRLS International Study Center, Lynch School of Education, Boston College.


## Appendix A

## Mathematics Cognitive Domains Framework: TIMSS 2003 Developmental Project Fourth and Eighth Grades

To respond correctly to TIMSS test items, students need to be familiar with the mathematics content being assessed, but they also need to draw on a range of cognitive skills. The first domain, knowing facts, procedures, and concepts, covers what the student needs to know, while the second, applying knowledge and conceptual understanding, focuses on the ability of the student to apply what he or she knows to solve problems or answer questions. The third domain, reasoning, goes beyond the solution of routine problems to encompass unfamiliar situations, complex contexts, and multi-step problems.

## Knowing Facts, Procedures, and Concepts

Facility in using mathematics, or reasoning about mathematical situations, depends on mathematical knowledge and familiarity with mathematical concepts. The more relevant knowledge a student is able to recall and the wider the range of concepts he or she has understood, the greater the potential for engaging a wide range of problem-solving situations and for developing mathematical understanding.

Without access to a knowledge base that enables easy recall of the language and basic facts and conventions of number, symbolic representation, and spatial relations, students would find purposeful mathematical thinking impossible. Facts encompass the factual knowledge that provides the basic language of mathematics, and the essential mathematical facts and properties that form the foundation for mathematical thought.

Procedures form a bridge between more basic knowledge and the use of mathematics for solving routine problems, especially those encountered by many people in their daily lives. In essence a fluent use of procedures entails recall of sets of actions and how to carry them out. Students need to be efficient and accurate in using a variety of computational procedures and tools. They need to see that particular procedures can be used to solve entire classes of problems, not just individual problems.

Knowledge of concepts enables students to make connections between elements of knowledge that, at best, would otherwise be retained as isolated facts. It allows them to make extensions beyond their existing knowledge, judge the validity of mathematical statements and methods, and create mathematical representations.

This cognitive domain covers the following behaviors:

| Recall | Recall definitions; terminology; number properties; <br> geometric properties; and notation (e.g., $\mathrm{a} \times \mathrm{b}=\mathrm{ab}$, <br> $\mathrm{a}+\mathrm{a}+\mathrm{a}=3 \mathrm{a})$. |
| :--- | :--- |
| Recognize | Recognize mathematical objects, shapes, numbers <br> and expressions. Recognize mathematical entities <br> that are mathematically equivalent, e.g. areas of <br> parts of figures to represent fractions, equivalent |
| familiar fractions, decimals and percents; simple |  |
| algebraic expressions that represent a straightfor- |  |
| ward situation (eighth grade); different orientations |  |
| of simple geometric figures; and the nets of simple |  |
| geometric figures (eighth grade). |  |


| Compute | Carry out algorithmic procedures for,,$+- \times, \div$, <br> or a combination of these with whole numbers, <br> fractions, decimals and integers. Approximate <br> numbers to estimate computations. Carry out <br> routine algebraic procedures. |
| :--- | :--- |
| Retrieve | Retrieve information from graphs, tables or other <br> sources; read simple scales. |
| Measure | Use measuring instruments to draw lines, angles, <br> and shapes to given specifications; use units of mea- <br> surement appropriately; and estimate measures. |
| Know | Know concepts (e.g., place value; rounding; that <br> length, area and volume are conserved under <br> certain conditions; equal and unequal chance). |
| Classify/Order | Classify/group objects, shapes, numbers and <br> expressions according to common properties; <br> make correct decisions about class membership; |
| and order numbers and objects by attributes. |  |

## Applying Knowledge and Understanding

Problem solving is a central aim, and often means, of teaching school mathematics, and hence this and supporting skills (e.g., select, represent, model) feature prominently in the applying knowledge and conceptual understanding domain. In items aligned with this domain, students need to apply mathematical knowledge of facts, skills, and procedures or understanding of mathematical concepts to create representations and solve problems. Representation of ideas forms the core of mathematical thinking and communication, and the ability to create equivalent representations are fundamental to success in the subject.

The problem settings are more routine than those aligned with the reasoning domain. The routine problems will typically have been
standard in classroom exercises designed to provide practice in particular methods or techniques. Some of these problems will have been in words that set the problem situation in a quasi-real context. Though they range in difficulty, each of these types of "textbook" problems is expected to be sufficiently familiar to students that they will essentially involve selecting and applying learned procedures.

Problems may be set in real-life situations, or may be concerned with purely mathematical questions involving, for example, numeric or algebraic expressions, functions, equations, geometric figures, or statistical data sets. Therefore, problem solving is included not only in the applying knowledge and conceptual understanding domain, with emphasis on the more familiar and routine tasks, but also in the reasoning domain.

This cognitive domain covers the following behaviors:

| Select | Select an efficient/appropriate operation, method <br> or strategy for solving problems where there is a <br> known algorithm or method of solution. Select <br> simple algebraic expressions which represent <br> straightforward situations (fourth grade). Select <br> the nets of simple geometric figures (fourth grade). <br> Select appropriate algorithms or formulas. |
| :--- | :--- |
| Represent | Display mathematical information and data in <br> diagrams, tables, charts, or graphs, and gener- <br> ate equivalent representations for a given math- <br> ematical entity or relationship. |
| Model | Generate an appropriate model, such as an equa- <br> tion or diagram for solving a routine problem. |
| Implement | Follow and execute a set of mathematical <br> instructions. |

## Solve Routine Solve routine problems (i.e., problems similar to

 Problems those target students are likely to have encountered in class). For example, use geometric properties to solve problems; compare and match different representations of data (eighth grade) and use data from charts, tables, graphs, and maps to solve routine problems.
## Reasoning

Reasoning mathematically involves the capacity for logical, systematic thinking. It includes intuitive and inductive reasoning based on patterns and regularities that can be used to arrive at solutions to nonroutine problems. Non-routine problems are problems that are very likely to be unfamiliar to students. They make cognitive demands over and above those needed for solution of routine problems, even when the knowledge and skills required for their solution have been learned. Non-routine problems may be purely mathematical or may have reallife settings. Both types of items involve transfer of knowledge and skills to new situations, and interactions among reasoning skills are usually a feature. Problems requiring reasoning may do so in different ways, because of the novelty of the context or the complexity of the situation or because any solution to the problem must involve several steps, perhaps drawing on knowledge and understanding from different areas of mathematics.

Even though of the other behaviors listed within the reasoning domain are those that may be drawn on in thinking about and solving novel or complex problems, each by itself represents a valuable outcome of mathematics education, with the potential to influence learners' thinking more generally. For example, reasoning involves the ability to observe and make conjectures. It also involves making logical deductions based on specific assumptions and rules, and justifying results.

This cognitive domain covers the following behaviors:

| Analyze | Determine and describe or use relationships <br> between variables or objects in mathemati- <br> cal situations; use proportional reasoning <br> (fourth grade); decompose geometric figures <br> to simplify solving a problem; draw the net of <br> a given unfamiliar solid; visualize transforma- <br> tions of three-dimensional figures; compare <br> and match different representations of the <br> same data (fourth grade); and make valid <br> inferences from given information. |
| :--- | :--- |
| Generalize | Extend the domain to which the result of <br> mathematical thinking and problem solving is <br> applicable by restating results in more general <br> and more widely applicable terms. |
| Synthesize/ | Combine (various) mathematical procedures <br> to establish results, and combine results to |
| Integrate | produce a further result. Make connections <br> between different elements of knowledge and <br> related representations, and make linkages |
| between related mathematical ideas. |  |



## Appendix B

## Overview of Procedures TIMSS 2003 Developmental Project

## Process for Establishing the Mathematics Cognitive Domains for Scaling and Reporting

As explained in Chapter 1, developing reliable and valid achievement scales in the cognitive domains began with conducting a meeting of mathematics experts to examine the classification of the TIMSS 2003 items. Hosted by the IEA Secretariat in Amsterdam, 10 participants (see below) met in February 2005.

## Participants in Mathematics Expert Meeting

Amsterdam, February 2005
Khattab Mohammad Abu Lebdeh - Jordan
Yu-Hsien Chang - Chinese Taipei
Tandi Clausen-May - England
Robert Garden - New Zealand
Barbara Japelj - Slovenia
Michael Martin - TIMSS Study Director
Ina Mullis - TIMSS Study Director
Peter Nystrom - Sweden
David Robitaille - Canada
Graham Ruddock - England

Based on an iterative process of discussion and classification of items, the meeting participants worked with the four cognitive domains specified in the TIMSS 2003 Framework - knowing facts and procedures; using concepts; solving routine problems; and reasoning - to devise the three cognitive domains used as the basis for this report. Essentially, the "knowing facts and procedures" and the "using concepts" domains in the TIMSS 2003 Framework were combined, and then distinctions between the combined domain and solving routine problems were clarified. Finally, distinctions were clarified between these two domains and reasoning. This process led to the three domains - knowing facts, procedures, and concepts; applying knowledge and understanding; and reasoning (see Appendix A). (For the TIMSS 2007 Framework, the participating countries suggested that these be shortened to knowing, applying, and reasoning for both mathematics and science.)

Subsequent to the Amsterdam meeting, the cognitive domains devised for the developmental project were reviewed by the TIMSS 2007 Science and Mathematics Item Review Committee (SMIRC). Hosted by the National Foundation for Educational Research in England and Wales (the institution of the IEA Chair and the TIMSS 2007 Mathematics Coordinator), this meeting was held in April 2005 in London. In particular, the SMIRC mathematics experts endorsed reporting according to the three cognitive domains and worked to further refine and clarify the description of each domain (see below for participants).

## Mathematics Participants in TIMSS 2007 Science and Mathematics Item Review Committee Meeting

London, April 2005
Khattab Mohammad Abu Lebdeh - Jordan
Alka Arora - TIMSS Research Associate
Kiril Bankov - Bulgaria
Robert Garden - New Zealand
Liv Sissel Gronmo - Norway
Chen-yung Lin - Chinese Taipei
Mary Lindquist - United States
Ina Mullis - TIMSS Study Director
Graham Ruddock - TIMSS 2007 Mathematics Coordinator
Hanako Senuma - Japan

## Characteristics of Items Within Cognitive Domains

IEA's TIMSS \& PIRLS International Study Center (ISC) examined the spread of the items within the three domains according to item type (constructed-response or multiple-choice), content domain (algebra, geometry, etc.), and average difficulty (mean percent correct) to ensure there was sufficient coverage within each domain. As shown in Exhibit B.l, the classification resulted in a substantial number of items in each cognitive domain at both eighth grade (first page) and fourth grade (second page). Of the 194 items at the eighth grade, 65 were classified in the knowing cognitive domain, 93 in the applying cognitive domain, and 36 in the reasoning cognitive domain. Of the 159 items at the fourth grade, 58 were classified in the knowing cognitive domain, 63 in the applying cognitive domain, and 38 in the reasoning cognitive domain.

Within each cognitive domain, there was a very good spread of items in terms of item type (constructed-response or multiple-choice) at both eighth and fourth grades. Equivalent percentages of applying items were multiple-choice and constructed-response. As would

Number of Items by Item Type and Cognitive Domains

| Item Type | Cognitive Domains |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Knowing | Applying | Reasoning | Total |
| Constructed Response | 11 | 34 | 21 | 66 |
| Multiple Choice | 54 | 59 | 15 | 128 |
| Total | 65 | 93 | 36 | 194 |

Percent of Score Points by Item Type and Cognitive Domains

| Item Type | Cognitive Domains |  |  | Total <br> Score Points |
| :--- | :---: | :---: | :---: | :---: |
|  | Knowing | Applying | Reasoning |  |
| Constructed Response | $14 \%$ | $47 \%$ | $39 \%$ | 128 |
| Multiple Choice | $42 \%$ | $46 \%$ | $12 \%$ | 213 |
| Total | $31 \%$ | $46 \%$ | $23 \%$ |  |

Number of Items by Content Domain and Cognitive Domain

| Content Domain | Cognitive Domains |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Knowing | Applying | Reasoning | Total |
| Number | 21 | 31 | 5 | 57 |
| Algebra | 22 | 12 | 13 | 47 |
| Measurement | 7 | 22 | 2 | 31 |
| Geometry | 10 | 12 | 9 | 31 |
| Data | 5 | 16 | 7 | 28 |
| Total | 65 | 93 | 36 | 194 |

Percent of Score Points by Content Domain and Cognitive Domain

| Content Domain | Cognitive Domains |  |  | Total <br> Score Points |
| :--- | :---: | :---: | :---: | :---: |
|  | Knowing | Applying | Reasoning |  |
| Number | $35 \%$ | $55 \%$ | $10 \%$ | 53 |
| Algebra | $43 \%$ | $23 \%$ | $34 \%$ | 33 |
| Measurement | $21 \%$ | $73 \%$ | $6 \%$ | 33 |
| Geometry | $30 \%$ | $36 \%$ | $33 \%$ | 34 |
| Data | $15 \%$ | $53 \%$ | $32 \%$ | 213 |
| Total | $31 \%$ | $46 \%$ | $23 \%$ |  |

Mean Percent Correct by Content Domain and Cognitive Domain

| Item Difficulties <br> (Mean Percent Correct) | Cognitive Domains |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Knowing | Applying | Reasoning | Total |
| Number | $50 \%$ | $43 \%$ | $36 \%$ | $45 \%$ |
| Algebra | $49 \%$ | $45 \%$ | $29 \%$ | $42 \%$ |
| Measurement | $55 \%$ | $37 \%$ | $41 \%$ | $41 \%$ |
| Geometry | $51 \%$ | $50 \%$ | $36 \%$ | $46 \%$ |
| Data | $53 \%$ | $46 \%$ | $34 \%$ | $44 \%$ |
| Total | $50 \%$ | $43 \%$ | $33 \%$ | $44 \%$ |

Exhibit B.1: Characteristics of Items Within Cognitive Domains


Number of Items by Item Type and Cognitive Domains

| Item Type | Cognitive Domains |  |  | Total |
| :--- | :---: | :---: | :---: | :---: |
|  | Knowing | Applying | Reasoning |  |
| Constructed Response | 17 | 28 | 24 | 69 |
| Multiple Choice | 41 | 35 | 14 | 90 |
| Total | 58 | 63 | 38 | $1599^{1}$ |

Number of Items by Content Domain and Cognitive Domain

| Content Domain | Cognitive Domains |  |  | Total |
| :--- | :---: | :---: | :---: | :---: |
|  | Knowing | Applying | Reasoning |  |
| Number | 25 | 19 | 19 | 63 |
| Patterns and Relationships | 2 | 13 | 8 | 23 |
| Measurement | 10 | 18 | 4 | 32 |
| Geometry | 17 | 6 | 1 | 24 |
| Data | 4 | 7 | 6 | 17 |
| Total | 58 | 63 | 38 | 159 |

Percent of Score Points by Content Domain and Cognitive Domain

| Content Domain | Cognitive Domains |  |  | Total <br> Score Points |
| :--- | :---: | :---: | :---: | :---: |
|  | Knowing | Applying | Reasoning |  |
| Number | $37 \%$ | $29 \%$ | $34 \%$ | 24 |
| Patterns and Relationships | $8 \%$ | $54 \%$ | $38 \%$ | 32 |
| Measurement | $31 \%$ | $56 \%$ | $13 \%$ | 25 |
| Geometry | $72 \%$ | $24 \%$ | $4 \%$ | 17 |
| Data | $24 \%$ | $41 \%$ | $35 \%$ | $\mathbf{1 6 2}$ |
| Total | $36 \%$ | $39 \%$ | $26 \%$ |  |

Mean Percent Correct by Content Domain and Cognitive Domain

| Item Difficulties <br> (Mean Percent Correct) | Cognitive Domains |  |  | Total |
| :--- | :---: | :---: | :---: | :---: |
|  | Knowing | Applying | Reasoning |  |
| Number | $63 \%$ | $56 \%$ | $37 \%$ | 53 |
| Patterns and Relationships | $63 \%$ | $57 \%$ | $36 \%$ | 50 |
| Measurement | $65 \%$ | $47 \%$ | $39 \%$ | 52 |
| Geometry | $60 \%$ | $53 \%$ | $43 \%$ | 57 |
| Data | $69 \%$ | $56 \%$ | $58 \%$ | 60 |
| Total | $53 \%$ | $53 \%$ | $40 \%$ | $\mathbf{5 4}$ |

1 There were 161 items on the fourth grade mathematics assessment. Following item review, two items were deleted, and were not included in the cognitive domain scaling.
be expected, however, at both grades a relatively higher percentage of items in the knowing domain were multiple-choice, and a commensurately higher percentage of items in the reasoning domain were constructed-response. Often, the multiple-choice format is a cost-effective way to assess specific knowledge, while the constructed-response format may be required in complex problem-solving situations involving multiple strategies.

Despite some unevenness, there was good spread across content domains within each of the three cognitive domains. At eighth grade, it would have been preferable to have a higher proportion of number items in the reasoning domain (an effort is being made to address this in TIMSS 2007). That the distribution for measurement is concentrated in the applying domain makes some sense, since by eighth grade students should know about basic measurement tools and units. (In the TIMSS 2007 Framework, aspects of measurement were incorporated into the number and geometry content domains because there is little emphasis on measurement in eighth-grade mathematics curricula around the world).

Because algebra is generally not taught as a formal subject in primary school, only introductory concepts about patterns and relationships are assessed at the fourth grade. As such, a higher proportion of patterns and relationship items in the knowing category would have been preferable at the fourth grade. (In the TIMSS 2007 Framework, the patterns and relationships content domain has been incorporated into the number content domain.) Also, a higher proportion of measurement items in the reasoning domain would have been better. The low coverage of geometry in the reasoning domain is understandable, since this is a subject little emphasized at the fourth grade. (In the TIMSS 2007 Framework, the geometry content domain, now called geometric shapes and measures, has been recast to better describe the fourth-grade curricula of participating countries.)

Finally, Exhibit B.l also shows a good range in item difficulty (mean percentage correct) internationally, on average, within each of
the three cognitive domains. As would be anticipated, at both grades there was the same overall pattern, with the reasoning items the most difficult. Essentially, this pattern of the items in the reasoning domain being more difficult than the knowing or applying items was consistent across the content domains at both grades.

## Constructing Achievement Scales in the Mathematics Cognitive Domains

The scaling methodology was identical to that used to report mathematics achievement results and achievement in the mathematics content domains in the TIMSS 2003 International Report. It is described in detail in Gonzalez, Galia, and Li (2004).

The TIMSS 2003 goals of broad coverage of the mathematics and science curriculum and of measuring trends across assessments necessitated a complex matrix-sampling booklet design, with individual students responding to a subset of the mathematics and science items in the assessment but not the entire assessment item pool. Given the complexities of the data collection and the need to have student scores on the entire assessment for analysis and reporting purposes, TIMSS 2003 relied on Item Response Theory (IRT) scaling to describe student achievement on the assessment and to provide accurate measures of trends from previous assessments. The TIMSS IRT scaling approach used multiple imputation, or "plausible values" methodology, to obtain proficiency scores in mathematics and science for all students, even though each student responded to only a part of the assessment item pool. To enhance the reliability of the student scores, the TIMSS scaling combined student responses to the items they were administered with information about students' backgrounds, a process known as "conditioning."

Using routine TIMSS procedures, three distinct IRT scaling models, depending on item type and scoring procedure, were used in constructing achievement scales for the mathematics cognitive domains. Each scaling model is a "latent variable" model that describes
the probability that a student will respond in a specific way to an item in terms of the respondent's proficiency, which is an unobserved or "latent" trait, and various characteristics (or "parameters") of the item. A three-parameter model was used with multiple-choice items, which were scored as correct or incorrect, and a two-parameter model for constructed-response items with just two response options, which also were scored as correct or incorrect. Since each of these item types has just two response categories, they are known as dichotomous items. A partial credit model was used with polytomous constructed-response items, i.e., those with more than two score points.

## Item Calibration

The first step in constructing the cognitive domain scales was to estimate the IRT model parameters for each item on each of the cognitive domain scales. This procedure, known as item calibration, was implemented using the PARSCALE software applied to a self-weighting random sample of 1000 students from each country's TIMSS 2003 student sample. Using student samples of equal size ensured that the data from each country contributed equally to the item calibration, while keeping the amount of data to be analyzed to a reasonable size.

At the fourth and eighth grades, separate calibrations were conducted for each of the three mathematics cognitive domains: knowing, applying, and reasoning (abbreviated labels). At the eighth grade, the calibrations were based on 46,000 student records; 1,000 from each of the 46 countries that participated in the 2003 assessment. At the fourth grade, the calibrations were based on 26,000 student records, 1,000 from each of the 26 countries that participated in the 2003 assessment at the fourth grade.

## Evaluating the Fit of the IRT Models

After the calibrations were completed, checks were performed to verify that the item parameters obtained from PARSCALE for the three cognitive scales were a good fit for the data. An item is said to fit the IRT model when the empirical distribution of student responses (i.e., the proportion of correct student responses at various levels of student proficiency) closely matches the theoretical item response curve constructed from the estimated item parameters. For every item at both grades, the empirical and theoretical distributions were plotted and compared.

## Generating IRT Proficiency Scores

Following item calibration, Educational Testing Service's MGROUP program was used to generate the IRT proficiency scores for the cognitive domain scales. This program takes as input the students' responses to the items they were given, the item parameters estimated at the calibration stage, and the conditioning variables derived from student background variables, and generates as output the plausible values that represent student proficiency.

Plausible values generated by the conditioning program are initially on the same scale as the item parameters used to estimate them. This scale metric is generally not useful for reporting purposes since it is somewhat arbitrary, ranges between approximately -3 and +3 , and has a mean of zero across all countries. The plausible values for each cognitive domain scale were transformed to the same metric as the overall mathematics scale in 2003, as was done for the content domain scaling in 2003. Thus, for the eighth grade, each of the three cognitive domain scales were set to have a mean of 467 and standard deviation of 100, and for the fourth grade, a mean of 495 and standard deviation of 100 .

## Reliability

Exhibit B. 2 displays the reliability coefficient for each country for the mathematics test overall and for the knowing, applying, and reasoning cognitive domains. The first page shows the reliabilities for the eighth grade and the second page shows the reliabilities for the fourth grade. Reliability was measured as the ratio of sampling variance to sampling variance plus imputation variance. This approach is more suitable for multiple-matrix-sampling designs where students respond to relatively few items than classical reliability methods (such as the well-known Kuder-Richardson formulas) that are affected by the number of items taken by the student. Reliability coefficients greater than .80 are generally considered acceptable for such designs.

At both grade levels, despite some variation, reliabilities generally were high for most countries. The international median (the median of the reliability coefficients for all countries) was .96 at the eighth grade and .97 at the fourth grade for the overall mathematics assessment. At the eighth grade, the median reliabilities for the cognitive domains were .93 for knowing, .96 for applying, and .88 for reasoning. At the fourth grade, they were .92 for knowing, .93 for applying, and .91 for reasoning.

| Countries | Reliabilities of Overall Mathematics and Cognitive Domains |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Overall | Knowing | Applying | Reasoning |
| Armenia | 0.97 | 0.96 | 0.92 | 0.93 |
| Australia | 0.99 | 0.97 | 0.99 | 0.97 |
| Bahrain | 0.83 | 0.51 | 0.72 | 0.46 |
| Belgium (Flemish) | 0.95 | 0.98 | 0.97 | 0.89 |
| Botswana | 0.72 | 0.67 | 0.73 | 0.56 |
| Bulgaria | 0.96 | 0.99 | 0.95 | 0.88 |
| Chile | 0.88 | 0.83 | 0.96 | 0.67 |
| Chinese Taipei | 0.96 | 0.99 | 0.96 | 0.92 |
| Cyprus | 0.79 | 0.33 | 0.91 | 0.90 |
| Egypt | 0.95 | 0.96 | 0.98 | 0.85 |
| England | 0.99 | 0.93 | 0.99 | 0.88 |
| Estonia | 0.96 | 0.95 | 0.96 | 0.87 |
| Ghana | 0.87 | 0.80 | 0.92 | 0.80 |
| Hong Kong, SAR | 0.95 | 0.88 | 0.97 | 1.00 |
| Hungary | 0.99 | 0.96 | 0.95 | 0.91 |
| Indonesia | 0.98 | 0.98 | 0.98 | 0.93 |
| Iran, Islamic Rep. of | 0.97 | 0.79 | 0.86 | 0.48 |
| Israel | 0.96 | 0.92 | 0.97 | 0.96 |
| Italy | 0.97 | 0.97 | 0.97 | 0.95 |
| Japan | 0.92 | 0.92 | 0.96 | 0.91 |
| Jordan | 0.99 | 0.93 | 0.97 | 0.72 |
| Korea, Rep. of | 0.71 | 0.82 | 0.67 | 0.79 |
| Latvia | 0.97 | 0.97 | 0.96 | 0.88 |
| Lebanon | 0.97 | 0.95 | 0.96 | 0.86 |
| Lithuania | 0.99 | 0.78 | 0.96 | 0.97 |
| Macedonia, Rep. of | 0.98 | 0.78 | 0.92 | 0.86 |
| Malaysia | 0.98 | 0.98 | 0.95 | 0.91 |
| Moldova, Rep. of | 0.98 | 0.88 | 0.96 | 0.93 |
| Morocco | 0.77 | 0.81 | 0.56 | 0.50 |
| Netherlands | 0.98 | 0.98 | 1.00 | 0.87 |
| New Zealand | 0.97 | 0.97 | 0.98 | 0.96 |
| Norway | 0.94 | 0.95 | 0.82 | 0.84 |
| Palestinian Nat'l Auth. | 0.96 | 0.90 | 0.85 | 0.95 |
| Philippines | 0.98 | 0.95 | 0.98 | 0.87 |
| Romania | 0.99 | 0.96 | 0.99 | 0.92 |
| Russian Federation | 0.91 | 0.93 | 0.96 | 0.82 |
| Saudi Arabia | 0.95 | 0.84 | 0.98 | 0.94 |
| Scotland | 0.93 | 0.92 | 0.97 | 0.99 |
| Serbia | 0.91 | 0.83 | 0.86 | 0.74 |
| Singapore | 0.96 | 0.98 | 0.97 | 0.97 |
| Slovak Republic | 0.96 | 0.97 | 0.92 | 0.92 |
| Slovenia | 0.91 | 0.68 | 0.78 | 0.74 |
| South Africa | 0.94 | 0.94 | 0.91 | 0.97 |
| Sweden | 0.95 | 0.93 | 0.91 | 0.84 |
| Tunisia | 0.94 | 0.72 | 0.83 | 0.52 |
| United States | 0.98 | 0.99 | 0.99 | 0.87 |
| International Median | 0.96 | 0.93 | 0.96 | 0.88 |
| Benchmark Participants: |  |  |  |  |
| Basque Country, Spain | 0.86 | 0.76 | 0.93 | 0.89 |
| Ontario Province, Can. | 0.95 | 0.89 | 0.97 | 0.97 |
| Quebec Province, Can. | 0.98 | 0.96 | 0.98 | 0.87 |
| Indiana State, US | 1.00 | 0.97 | 0.97 | 0.84 |


| Countries | Reliabilities of Overall Mathematics and Cognitive Domains |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Overall | Knowing | Applying | Reasoning |
| Armenia | 0.99 | 0.94 | 0.99 | 0.97 |
| Australia | 0.97 | 0.98 | 0.98 | 0.91 |
| Belgium (Flemish) | 0.99 | 0.83 | 0.76 | 0.91 |
| Chinese Taipei | 0.94 | 0.88 | 0.89 | 0.79 |
| Cyprus | 0.98 | 0.89 | 0.91 | 0.83 |
| England | 0.98 | 0.84 | 0.95 | 0.97 |
| Hong Kong, SAR | 0.95 | 0.95 | 0.93 | 0.91 |
| Hungary | 0.95 | 0.91 | 0.96 | 0.93 |
| Iran, Islamic Rep. of | 0.93 | 0.83 | 0.95 | 0.95 |
| Italy | 0.99 | 0.98 | 0.93 | 0.96 |
| Japan | 0.90 | 0.71 | 0.74 | 0.77 |
| Latvia | 0.97 | 0.97 | 0.87 | 0.90 |
| Lithuania | 0.99 | 0.99 | 0.88 | 0.94 |
| Moldova, Rep. of | 0.98 | 0.92 | 0.99 | 0.94 |
| Morocco | 0.95 | 0.93 | 0.98 | 0.91 |
| Netherlands | 0.91 | 0.92 | 0.79 | 0.59 |
| New Zealand | 0.99 | 0.88 | 0.87 | 0.80 |
| Norway | 0.97 | 0.89 | 0.92 | 0.87 |
| Philippines | 0.99 | 0.97 | 0.98 | 0.93 |
| Russian Federation | 0.99 | 0.95 | 1.00 | 0.99 |
| Scotland | 0.95 | 0.92 | 0.91 | 0.87 |
| Singapore | 0.99 | 0.95 | 1.00 | 1.00 |
| Slovenia | 0.97 | 0.80 | 0.86 | 0.98 |
| Tunisia | 0.93 | 0.88 | 0.93 | 0.91 |
| United States | 0.99 | 0.96 | 0.85 | 0.93 |
| International Median | 0.97 | 0.92 | 0.93 | 0.91 |
| Benchmark Participants: |  |  |  |  |
| Ontario Province, Can. | 0.98 | 0.95 | 0.99 | 0.98 |
| Quebec Province, Can. | 0.94 | 0.84 | 0.85 | 0.91 |
| Indiana State, US | 1.00 | 0.79 | 0.93 | 0.75 |

## Correlations

Exhibit B. 3 presents the Pearson correlation coefficient indicating the linear relationship between achievement in each cognitive domain and achievement on the overall mathematics assessment for each of the TIMSS 2003 countries. The first page shows the correlations for the eighth grade and the second page the correlations for the fourth grade. All of the correlations are substantial, indicating that high performance in each of the three cognitive domains is likely to be associated with high performance on the mathematics assessment overall. This means proficiency in each of the domains is an important contributor to mathematics proficiency in general.

At eighth grade, correlations were highest for knowing and applying, with a median correlation with overall mathematics achievement of .88 in each case. This means that students with high scores in these domains were equally likely to have high scores on mathematics overall. The correlation between reasoning and overall achievement was generally lower, with a median correlation of .77 (consistent with the somewhat lower reliability of the reasoning scale). This means that students with high scores in the reasoning domain also were likely to have high scores on mathematics overall, but somewhat less likely than students with high scores in the knowing or applying domains.

At the fourth grade, correlations between achievement in the cognitive domains and overall mathematics were more uniform, with correlations of .84 for the knowing domain, .86 for the applying domain, and .83 for the reasoning domain. This means that students with high scores in any one of the three cognitive domains were equally likely to have high scores on mathematics overall.

Correlations between the three cognitive scales are presented in Exhibit B. 4 for the eighth grade (first page) and for the fourth grade (second page). As would be expected of cognitive domains within a single subject area, mathematics, country-level correlations at the eighth grade were generally moderate to high, with international

| Countries | Pearson Correlations of Mathematics Cognitive Domains with Overall Mathematics |  |  |
| :---: | :---: | :---: | :---: |
|  | Knowing | Applying | Reasoning |
| Armenia | 0.85 | 0.87 | 0.79 |
| Australia | 0.89 | 0.90 | 0.81 |
| Bahrain | 0.81 | 0.81 | 0.69 |
| Belgium (Flemish) | 0.91 | 0.91 | 0.83 |
| Botswana | 0.75 | 0.76 | 0.64 |
| Bulgaria | 0.88 | 0.88 | 0.76 |
| Chile | 0.85 | 0.85 | 0.71 |
| Chinese Taipei | 0.92 | 0.93 | 0.86 |
| Cyprus | 0.86 | 0.87 | 0.76 |
| Egypt | 0.84 | 0.84 | 0.73 |
| England | 0.89 | 0.90 | 0.79 |
| Estonia | 0.88 | 0.89 | 0.79 |
| Ghana | 0.65 | 0.68 | 0.54 |
| Hong Kong, SAR | 0.89 | 0.90 | 0.82 |
| Hungary | 0.90 | 0.91 | 0.82 |
| Indonesia | 0.86 | 0.86 | 0.71 |
| Iran, Islamic Rep. of | 0.81 | 0.81 | 0.70 |
| Israel | 0.89 | 0.89 | 0.79 |
| Italy | 0.88 | 0.88 | 0.77 |
| Japan | 0.90 | 0.90 | 0.83 |
| Jordan | 0.86 | 0.86 | 0.76 |
| Korea, Rep. of | 0.90 | 0.91 | 0.83 |
| Latvia | 0.87 | 0.88 | 0.79 |
| Lebanon | 0.81 | 0.82 | 0.63 |
| Lithuania | 0.88 | 0.89 | 0.78 |
| Macedonia, Rep. of | 0.86 | 0.87 | 0.75 |
| Malaysia | 0.89 | 0.90 | 0.78 |
| Moldova, Rep. of | 0.85 | 0.85 | 0.74 |
| Morocco | 0.70 | 0.70 | 0.56 |
| Netherlands | 0.90 | 0.91 | 0.81 |
| New Zealand | 0.88 | 0.89 | 0.77 |
| Norway | 0.85 | 0.86 | 0.77 |
| Palestinian Nat'l Auth. | 0.82 | 0.83 | 0.72 |
| Philippines | 0.82 | 0.83 | 0.71 |
| Romania | 0.89 | 0.90 | 0.78 |
| Russian Federation | 0.88 | 0.89 | 0.76 |
| Saudi Arabia | 0.70 | 0.71 | 0.58 |
| Scotland | 0.89 | 0.90 | 0.79 |
| Serbia | 0.89 | 0.89 | 0.80 |
| Singapore | 0.92 | 0.92 | 0.85 |
| Slovak Republic | 0.89 | 0.90 | 0.82 |
| Slovenia | 0.87 | 0.88 | 0.75 |
| South Africa | 0.82 | 0.83 | 0.70 |
| Sweden | 0.87 | 0.87 | 0.75 |
| Tunisia | 0.74 | 0.75 | 0.57 |
| United States | 0.91 | 0.91 | 0.82 |
| International Median | 0.88 | 0.88 | 0.77 |
| Benchmark Participants: |  |  |  |
| Basque Country, Spain | 0.83 | 0.84 | 0.71 |
| Ontario Province, Can. | 0.86 | 0.88 | 0.74 |
| Quebec Province, Can. | 0.84 | 0.87 | 0.74 |
| Indiana State, US | 0.81 | 0.82 | 0.73 |


| Countries | Pearson Correlations of Mathematics Cognitive Domains with Overall Mathematics |  |  |
| :---: | :---: | :---: | :---: |
|  | Knowing | Applying | Reasoning |
| Armenia | 0.81 | 0.84 | 0.77 |
| Australia | 0.86 | 0.87 | 0.84 |
| Belgium (Flemish) | 0.80 | 0.83 | 0.78 |
| Chinese Taipei | 0.82 | 0.84 | 0.81 |
| Cyprus | 0.85 | 0.88 | 0.84 |
| England | 0.87 | 0.89 | 0.85 |
| Hong Kong, SAR | 0.81 | 0.84 | 0.81 |
| Hungary | 0.85 | 0.88 | 0.83 |
| Iran, Islamic Rep. of | 0.78 | 0.80 | 0.71 |
| Italy | 0.86 | 0.88 | 0.83 |
| Japan | 0.83 | 0.86 | 0.82 |
| Latvia | 0.84 | 0.87 | 0.83 |
| Lithuania | 0.85 | 0.87 | 0.83 |
| Moldova, Rep. of | 0.85 | 0.88 | 0.83 |
| Morocco | 0.72 | 0.74 | 0.63 |
| Netherlands | 0.77 | 0.82 | 0.76 |
| New Zealand | 0.87 | 0.88 | 0.86 |
| Norway | 0.82 | 0.85 | 0.79 |
| Philippines | 0.82 | 0.83 | 0.77 |
| Russian Federation | 0.85 | 0.88 | 0.85 |
| Scotland | 0.84 | 0.86 | 0.81 |
| Singapore | 0.85 | 0.89 | 0.87 |
| Slovenia | 0.84 | 0.86 | 0.83 |
| Tunisia | 0.75 | 0.77 | 0.66 |
| United States | 0.85 | 0.88 | 0.85 |
| International Median | 0.84 | 0.86 | 0.83 |
| Benchmark Participants: |  |  |  |
| Ontario Province, Can. | 0.84 | 0.86 | 0.83 |
| Quebec Province, Can. | 0.82 | 0.84 | 0.80 |
| Indiana State, US | 0.77 | 0.79 | 0.75 |


| Countries | Pearson Correlations for Mathematics Cognitive Domains |  |  |
| :---: | :---: | :---: | :---: |
|  | Knowing <br> Applying | Knowing Reasoning | Applying Reasoning |
| Armenia | 0.91 | 0.79 | 0.86 |
| Australia | 0.95 | 0.83 | 0.84 |
| Bahrain | 0.91 | 0.75 | 0.78 |
| Belgium (Flemish) | 0.95 | 0.85 | 0.85 |
| Botswana | 0.89 | 0.74 | 0.76 |
| Bulgaria | 0.95 | 0.80 | 0.80 |
| Chile | 0.94 | 0.78 | 0.76 |
| Chinese Taipei | 0.97 | 0.89 | 0.88 |
| Cyprus | 0.95 | 0.82 | 0.80 |
| Egypt | 0.95 | 0.81 | 0.82 |
| England | 0.95 | 0.80 | 0.82 |
| Estonia | 0.94 | 0.82 | 0.82 |
| Ghana | 0.74 | 0.60 | 0.62 |
| Hong Kong, SAR | 0.95 | 0.85 | 0.84 |
| Hungary | 0.95 | 0.85 | 0.84 |
| Indonesia | 0.94 | 0.76 | 0.77 |
| Iran, Islamic Rep. of | 0.92 | 0.77 | 0.78 |
| Israel | 0.95 | 0.83 | 0.81 |
| Italy | 0.95 | 0.80 | 0.79 |
| Japan | 0.96 | 0.86 | 0.86 |
| Jordan | 0.94 | 0.84 | 0.82 |
| Korea, Rep. of | 0.96 | 0.86 | 0.85 |
| Latvia | 0.94 | 0.82 | 0.82 |
| Lebanon | 0.91 | 0.67 | 0.68 |
| Lithuania | 0.95 | 0.81 | 0.81 |
| Macedonia, Rep. of | 0.95 | 0.78 | 0.82 |
| Malaysia | 0.96 | 0.81 | 0.83 |
| Moldova, Rep. of | 0.95 | 0.79 | 0.79 |
| Morocco | 0.86 | 0.65 | 0.71 |
| Netherlands | 0.94 | 0.81 | 0.84 |
| New Zealand | 0.94 | 0.79 | 0.79 |
| Norway | 0.93 | 0.82 | 0.81 |
| Palestinian Nat'l Auth. | 0.93 | 0.81 | 0.79 |
| Philippines | 0.93 | 0.79 | 0.80 |
| Romania | 0.95 | 0.82 | 0.81 |
| Russian Federation | 0.94 | 0.77 | 0.78 |
| Saudi Arabia | 0.80 | 0.66 | 0.65 |
| Scotland | 0.95 | 0.82 | 0.81 |
| Serbia | 0.95 | 0.84 | 0.84 |
| Singapore | 0.96 | 0.88 | 0.86 |
| Slovak Republic | 0.95 | 0.85 | 0.85 |
| Slovenia | 0.94 | 0.77 | 0.78 |
| South Africa | 0.89 | 0.76 | 0.78 |
| Sweden | 0.93 | 0.76 | 0.74 |
| Tunisia | 0.87 | 0.65 | 0.68 |
| United States | 0.97 | 0.86 | 0.85 |
| International Median | 0.95 | 0.81 | 0.81 |
| Benchmark Participants: |  |  |  |
| Basque Country, Spain | 0.93 | 0.78 | 0.75 |
| Ontario Province, Can. | 0.92 | 0.75 | 0.76 |
| Quebec Province, Can. | 0.92 | 0.75 | 0.76 |
| Indiana State, US | 0.93 | 0.82 | 0.80 |

## Exhibit B.4: Correlations of Mathematics Cognitive Domains

| Countries | Pearson Correlations for Mathematics Cognitive Domains |  |  |
| :---: | :---: | :---: | :---: |
|  | Knowing <br> Applying | Knowing Reasoning | Applying Reasoning |
| Armenia | 0.84 | 0.74 | 0.86 |
| Australia | 0.92 | 0.89 | 0.91 |
| Belgium (Flemish) | 0.89 | 0.80 | 0.84 |
| Chinese Taipei | 0.92 | 0.87 | 0.91 |
| Cyprus | 0.92 | 0.87 | 0.91 |
| England | 0.94 | 0.89 | 0.91 |
| Hong Kong, SAR | 0.91 | 0.85 | 0.90 |
| Hungary | 0.90 | 0.82 | 0.89 |
| Iran, Islamic Rep. of | 0.86 | 0.73 | 0.81 |
| Italy | 0.92 | 0.84 | 0.88 |
| Japan | 0.91 | 0.84 | 0.89 |
| Latvia | 0.91 | 0.85 | 0.88 |
| Lithuania | 0.93 | 0.86 | 0.90 |
| Moldova, Rep. of | 0.89 | 0.82 | 0.89 |
| Morocco | 0.80 | 0.63 | 0.74 |
| Netherlands | 0.87 | 0.80 | 0.85 |
| New Zealand | 0.93 | 0.88 | 0.90 |
| Norway | 0.92 | 0.80 | 0.86 |
| Philippines | 0.90 | 0.83 | 0.86 |
| Russian Federation | 0.88 | 0.85 | 0.90 |
| Scotland | 0.91 | 0.85 | 0.87 |
| Singapore | 0.92 | 0.86 | 0.94 |
| Slovenia | 0.91 | 0.87 | 0.92 |
| Tunisia | 0.80 | 0.69 | 0.73 |
| United States | 0.93 | 0.88 | 0.92 |
| International Median | 0.91 | 0.85 | 0.89 |
| Benchmark Participants: |  |  |  |
| Ontario Province, Can. | 0.91 | 0.87 | 0.90 |
| Quebec Province, Can. | 0.91 | 0.83 | 0.86 |
| Indiana State, US | 0.90 | 0.83 | 0.88 |

medians of .95 between the knowing and applying domains, 81 between the knowing and reasoning domains, and .81 between the applying and reasoning domains. The highest correlation was between the knowing and applying domains, which makes sense considering that these were the two domains with the highest correlation with mathematics achievement overall.

At the fourth grade, country-level correlations between the cognitive domains also were high, with international medians of . 91 between the knowing and applying domains, .85 between the knowing and reasoning domains, and .89 between the applying and reasoning domains. The relatively large correlations between the cognitive domain scales show that student performance in the cognitive domains is not independent, and that high-scoring students on one scale are likely also to be high scorers on another. Despite the high correlations, however, there is scope for interesting average score differences between countries on the three cognitive scales.


## Appendix C

## Coverage of TIMSS 2003 Target Populations and Participation Rates

Exhibit C.1: Coverage of TIMSS 2003 Target Populations


| Countries | International Desired Population |  | National Desired Population |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Coverage | Notes on Coverage | School-Level Exclusions | Within-Sample Exclusions | Overall Exclusions |
| Armenia | 100\% |  | 2.9\% | 0.0\% | 2.9\% |
| Australia | 100\% |  | 0.4\% | 0.9\% | 1.3\% |
| Bahrain | 100\% |  | 0.0\% | 0.0\% | 0.0\% |
| Belgium (Flemish) | 100\% |  | 3.1\% | 0.1\% | 3.2\% |
| Botswana | 100\% |  | 0.8\% | 2.2\% | 3.0\% |
| Bulgaria | 100\% |  | 0.5\% | 0.0\% | 0.5\% |
| Chile | 100\% |  | 1.6\% | 0.7\% | 2.2\% |
| Chinese Taipei | 100\% |  | 0.2\% | 4.6\% | 4.8\% |
| Cyprus | 100\% |  | 1.1\% | 1.5\% | 2.5\% |
| Egypt | 100\% |  | 3.4\% | 0.0\% | 3.4\% |
| England | 100\% |  | 2.1\% | 0.0\% | 2.1\% |
| Estonia | 100\% |  | 2.6\% | 0.8\% | 3.4\% |
| Ghana | 100\% |  | 0.9\% | 0.0\% | 0.9\% |
| Hong Kong, SAR | 100\% |  | 3.3\% | 0.1\% | 3.4\% |
| Hungary | 100\% |  | 5.5\% | 3.2\% | 8.5\% |
| Indonesia | 80\% | Non-Islamic schools | 0.1\% | 0.3\% | 0.4\% |
| Iran, Islamic Rep. of | 100\% |  | 5.5\% | 1.1\% | 6.5\% |
| Israel | 100\% |  | 15.2\% | 8.6\% | 22.5\% |
| Italy | 100\% |  | 0.0\% | 3.6\% | 3.6\% |
| Japan | 100\% |  | 0.5\% | 0.1\% | 0.6\% |
| Jordan | 100\% |  | 0.5\% | 0.8\% | 1.3\% |
| Korea, Rep. of | 100\% |  | 1.5\% | 3.4\% | 4.9\% |
| Latvia | 100\% |  | 3.6\% | 0.1\% | 3.7\% |
| Lebanon | 100\% |  | 1.4\% | 0.0\% | 1.4\% |
| Lithuania | 89\% | Students taught in Lithuanian | 1.4\% | 1.2\% | 2.6\% |
| Macedonia, Rep. of | 100\% |  | 12.5\% | 0.0\% | 12.5\% |
| Malaysia | 100\% |  | 4.0\% | 0.0\% | 4.0\% |
| Moldova, Rep. of | 100\% |  | 0.7\% | 0.5\% | 1.2\% |
| Morocco | 69\% | All students but Souss Massa Draa, Casablanca, Gharb-Chrarda | 1.5\% | 0.0\% | 1.5\% |
| Netherlands | 100\% |  | 3.0\% | 0.0\% | 3.0\% |
| New Zealand | 100\% |  | 1.7\% | 2.7\% | 4.4\% |
| Norway | 100\% |  | 0.9\% | 1.5\% | 2.3\% |
| Palestinian Nat'l Auth. | 100\% |  | 0.2\% | 0.3\% | 0.5\% |
| Philippines | 100\% |  | 1.5\% | 0.0\% | 1.5\% |
| Romania | 100\% |  | 0.4\% | 0.1\% | 0.5\% |
| Russian Federation | 100\% |  | 1.7\% | 3.9\% | 5.5\% |
| Saudi Arabia | 100\% |  | 0.3\% | 0.2\% | 0.5\% |
| Scotland | 100\% |  | 0.0\% | 0.0\% | 0.0\% |
| Serbia | 81\% | Serbia without Kosovo | 2.4\% | 0.6\% | 2.9\% |
| Singapore | 100\% |  | 0.0\% | 0.0\% | 0.0\% |
| Slovak Republic | 100\% |  | 5.0\% | 0.0\% | 5.0\% |
| Slovenia | 100\% |  | 1.3\% | 0.1\% | 1.4\% |
| South Africa | 100\% |  | 0.6\% | 0.0\% | 0.6\% |
| Sweden | 100\% |  | 0.3\% | 2.5\% | 2.8\% |
| Syrian Arab Republic | 100\% |  | 18.7\% | 0.0\% | 18.8\% |
| Tunisia | 100\% |  | 1.8\% | 0.0\% | 1.8\% |
| United States | 100\% |  | 0.0\% | 4.9\% | 4.9\% |
| Benchmarking Participants |  |  |  |  |  |
| Basque Region, Spain | 100\% |  | 2.1\% | 3.8\% | 5.8\% |
| Indiana State, US | 100\% |  | 0.0\% | 7.8\% | 7.8\% |
| Ontario Province, Can. | 100\% |  | 1.0\% | 5.0\% | 6.0\% |
| Quebec Province, Can. | 100\% |  | 1.4\% | 3.5\% | 4.8\% |

## Exhibit C.1: Coverage of TIMSS 2003 Target Populations

| Countries | International Desired Population |  | National Desired Population |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Coverage | Notes on Coverage | School-Level Exclusions | Within-Sample Exclusions | Overall Exclusions |
| Armenia | 100\% |  | 2.9\% | 0.0\% | 2.9\% |
| Australia | 100\% |  | 1.2\% | 1.6\% | 2.7\% |
| Belgium (Flemish) | 100\% |  | 5.9\% | 0.4\% | 6.3\% |
| Chinese Taipei | 100\% |  | 0.3\% | 2.8\% | 3.1\% |
| Cyprus | 100\% |  | 1.5\% | 1.4\% | 2.9\% |
| England | 100\% |  | 1.9\% | 0.0\% | 1.9\% |
| Hong Kong, SAR | 100\% |  | 3.7\% | 0.1\% | 3.8\% |
| Hungary | 100\% |  | 4.4\% | 3.9\% | 8.1\% |
| Iran, Islamic Rep. of | 100\% |  | 3.6\% | 2.1\% | 5.7\% |
| Italy | 100\% |  | 0.1\% | 4.1\% | 4.2\% |
| Japan | 100\% |  | 0.4\% | 0.3\% | 0.8\% |
| Latvia | 100\% |  | 4.3\% | 0.1\% | 4.4\% |
| Lithuania | 92\% | Students taught in Lithuanian | 2.1\% | 2.6\% | 4.6\% |
| Moldova, Rep. of | 100\% |  | 2.0\% | 1.6\% | 3.6\% |
| Morocco | 100\% |  | 2.2\% | 0.0\% | 2.2\% |
| Netherlands | 100\% |  | 4.1\% | 1.1\% | 5.2\% |
| New Zealand | 100\% |  | 1.5\% | 2.5\% | 4.0\% |
| Norway | 100\% |  | 1.7\% | 2.7\% | 4.4\% |
| Philippines | 100\% |  | 3.8\% | 0.7\% | 4.5\% |
| Russian Federation | 100\% |  | 2.2\% | 4.7\% | 6.8\% |
| Scotland | 100\% |  | 1.5\% | 0.0\% | 1.5\% |
| Singapore | 100\% |  | 0.0\% | 0.0\% | 0.0\% |
| Slovenia | 100\% |  | 0.8\% | 0.5\% | 1.3\% |
| Tunisia | 100\% |  | 0.9\% | 0.0\% | 0.9\% |
| United States | 100\% |  | 0.0\% | 5.1\% | 5.1\% |
| Yemen | 100\% |  | 0.6\% | 8.9\% | 9.5\% |
| Benchmarking Participants |  |  |  |  |  |
| Indiana State, US | 100\% |  | 0.0\% | 7.2\% | 7.2\% |
| Ontario Province, Can. | 100\% |  | 1.3\% | 3.5\% | 4.8\% |
| Quebec Province, Can. | 100\% |  | 2.7\% | 0.9\% | 3.6\% |


| Countries | School Participation |  | Class <br> Participation | Student Participation | Overall Participation |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Before Replacement | After Replacement |  |  | Before Replacement | After Replacement |
| Armenia | 99\% | 99\% | 99\% | 90\% | 89\% | 89\% |
| Australia | 81\% | 90\% | 100\% | 93\% | 75\% | 83\% |
| Bahrain | 100\% | 100\% | 100\% | 98\% | 98\% | 98\% |
| Belgium (Flemish) | 82\% | 99\% | 98\% | 97\% | 77\% | 94\% |
| Botswana | 98\% | 98\% | 100\% | 98\% | 96\% | 96\% |
| Bulgaria | 97\% | 97\% | 99\% | 96\% | 92\% | 92\% |
| Chile | 98\% | 100\% | 100\% | 99\% | 97\% | 99\% |
| Chinese Taipei | 100\% | 100\% | 100\% | 99\% | 99\% | 99\% |
| Cyprus | 100\% | 100\% | 100\% | 96\% | 96\% | 96\% |
| Egypt | 99\% | 100\% | 100\% | 97\% | 97\% | 97\% |
| England | 40\% | 54\% | 99\% | 86\% | 34\% | 46\% |
| Estonia | 99\% | 99\% | 100\% | 96\% | 95\% | 95\% |
| Ghana | 100\% | 100\% | 100\% | 93\% | 93\% | 93\% |
| Hong Kong, SAR | 74\% | 83\% | 99\% | 97\% | 72\% | 80\% |
| Hungary | 98\% | 99\% | 100\% | 95\% | 94\% | 94\% |
| Indonesia | 98\% | 100\% | 100\% | 99\% | 97\% | 99\% |
| Iran, Islamic Rep. of | 100\% | 100\% | 100\% | 98\% | 98\% | 98\% |
| Israel | 98\% | 99\% | 100\% | 95\% | 93\% | 94\% |
| Italy | 96\% | 100\% | 100\% | 97\% | 93\% | 97\% |
| Japan | 97\% | 97\% | 100\% | 96\% | 93\% | 93\% |
| Jordan | 100\% | 100\% | 100\% | 96\% | 96\% | 96\% |
| Korea, Rep. of | 99\% | 99\% | 100\% | 99\% | 98\% | 98\% |
| Latvia | 92\% | 94\% | 100\% | 89\% | 81\% | 83\% |
| Lebanon | 93\% | 95\% | 100\% | 96\% | 89\% | 91\% |
| Lithuania | 92\% | 95\% | 100\% | 89\% | 81\% | 84\% |
| Macedonia, Rep. of | 94\% | 99\% | 100\% | 97\% | 91\% | 96\% |
| Malaysia | 100\% | 100\% | 100\% | 98\% | 98\% | 98\% |
| Moldova, Rep. of | 99\% | 100\% | 100\% | 96\% | 95\% | 96\% |
| Morocco | 79\% | 79\% | 100\% | 91\% | 71\% | 71\% |
| Netherlands | 79\% | 87\% | 100\% | 94\% | 74\% | 81\% |
| New Zealand | 86\% | 97\% | 100\% | 93\% | 80\% | 90\% |
| Norway | 92\% | 92\% | 100\% | 92\% | 85\% | 85\% |
| Palestinian Nat'l Auth. | 100\% | 100\% | 100\% | 99\% | 99\% | 99\% |
| Philippines | 81\% | 86\% | 100\% | 96\% | 78\% | 82\% |
| Romania | 99\% | 99\% | 100\% | 98\% | 98\% | 98\% |
| Russian Federation | 99\% | 99\% | 100\% | 97\% | 96\% | 96\% |
| Saudi Arabia | 95\% | 97\% | 100\% | 97\% | 93\% | 94\% |
| Scotland | 76\% | 85\% | 100\% | 89\% | 68\% | 76\% |
| Serbia | 99\% | 99\% | 100\% | 96\% | 96\% | 96\% |
| Singapore | 100\% | 100\% | 100\% | 97\% | 97\% | 97\% |
| Slovak Republic | 96\% | 100\% | 100\% | 95\% | 91\% | 95\% |
| Slovenia | 94\% | 99\% | 100\% | 93\% | 87\% | 91\% |
| South Africa | 89\% | 96\% | 100\% | 92\% | 82\% | 88\% |
| Sweden | 97\% | 99\% | 99\% | 89\% | 85\% | 87\% |
| Syrian Arab Republic | 81\% | 89\% | 100\% | 98\% | 79\% | 87\% |
| Tunisia | 100\% | 100\% | 100\% | 98\% | 98\% | 98\% |
| United States | 71\% | 78\% | 99\% | 94\% | 66\% | 73\% |
| Benchmarking Participants |  |  |  |  |  |  |
| Basque Region, Spain | 100\% | 100\% | 100\% | 98\% | 97\% | 98\% |
| Indiana State, US | 97\% | 97\% | 100\% | 97\% | 94\% | 94\% |
| Ontario Province, Can. | 84\% | 93\% | 100\% | 95\% | 80\% | 89\% |
| Quebec Province, Can. | 91\% | 93\% | 100\% | 92\% | 84\% | 85\% |

## Exhibit C.2: Participation Rates (Weighted)

| Countries | School Participation |  | Class <br> Participation | Student Participation | Overall Participation |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Before Replacement | After <br> Replacement |  |  | Before Replacement | After <br> Replacement |
| Armenia | 99\% | 99\% | 100\% | 91\% | 90\% | 90\% |
| Australia | 78\% | 90\% | 100\% | 94\% | 73\% | 85\% |
| Belgium (Flemish) | 89\% | 99\% | 100\% | 98\% | 87\% | 97\% |
| Chinese Taipei | 100\% | 100\% | 100\% | 99\% | 99\% | 99\% |
| Cyprus | 100\% | 100\% | 100\% | 97\% | 97\% | 97\% |
| England | 54\% | 82\% | 100\% | 93\% | 50\% | 76\% |
| Hong Kong, SAR | 77\% | 88\% | 99\% | 95\% | 73\% | 83\% |
| Hungary | 98\% | 99\% | 100\% | 94\% | 92\% | 93\% |
| Iran, Islamic Rep. of | 100\% | 100\% | 100\% | 98\% | 98\% | 98\% |
| Italy | 97\% | 100\% | 100\% | 97\% | 93\% | 97\% |
| Japan | 100\% | 100\% | 100\% | 97\% | 97\% | 97\% |
| Latvia | 91\% | 94\% | 100\% | 94\% | 85\% | 88\% |
| Lithuania | 92\% | 96\% | 99\% | 92\% | 84\% | 87\% |
| Moldova, Rep. of | 97\% | 100\% | 100\% | 97\% | 94\% | 97\% |
| Morocco | 87\% | 87\% | 100\% | 93\% | 81\% | 81\% |
| Netherlands | 52\% | 87\% | 100\% | 96\% | 50\% | 84\% |
| New Zealand | 87\% | 98\% | 100\% | 95\% | 82\% | 93\% |
| Norway | 89\% | 93\% | 100\% | 95\% | 85\% | 88\% |
| Philippines | 78\% | 85\% | 100\% | 95\% | 75\% | 81\% |
| Russian Federation | 99\% | 100\% | 100\% | 97\% | 96\% | 97\% |
| Scotland | 64\% | 83\% | 100\% | 92\% | 59\% | 77\% |
| Singapore | 100\% | 100\% | 100\% | 98\% | 98\% | 98\% |
| Slovenia | 95\% | 99\% | 100\% | 92\% | 87\% | 91\% |
| Tunisia | 100\% | 100\% | 100\% | 99\% | 99\% | 99\% |
| United States | 70\% | 82\% | 99\% | 95\% | 66\% | 78\% |
| Yemen | 100\% | 100\% | 100\% | 93\% | 93\% | 93\% |
| Benchmarking Participants |  |  |  |  |  |  |
| Indiana State, US | 100\% | 100\% | 100\% | 98\% | 98\% | 98\% |
| Ontario Province, Can. | 89\% | 94\% | 100\% | 96\% | 85\% | 90\% |
| Quebec Province, Can. | 99\% | 100\% | 100\% | 91\% | 90\% | 91\% |



Appendix D
Percentiles and Standard Deviations of Mathematics Achievement in the Cognitive Domains

| Countries | 5th Percentile | 25th <br> Percentile | 50th Percentile | $\begin{aligned} & \text { 75th } \\ & \text { Percentile } \end{aligned}$ | $\begin{aligned} & \text { 95th } \\ & \text { Percentile } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Armenia | 347 (4.9) | 429 (3.4) | 484 (2.9) | 535 (4.2) | 600 (3.7) |
| Australia | 381 (7.8) | 451 (3.2) | 497 (5.2) | 544 (4.8) | 609 (7.8) |
| Bahrain | 265 (4.9) | 342 (2.4) | 401 (3.1) | 459 (2.6) | 538 (5.5) |
| Belgium (Flemish) | 405 (6.9) | 499 (3.6) | 546 (2.6) | 585 (3.5) | 634 (4.9) |
| Botswana | 251 (5.0) | 321 (2.9) | 373 (2.3) | 423 (3.4) | 492 (5.4) |
| Bulgaria | 352 (8.5) | 431 (4.9) | 486 (5.2) | 542 (5.2) | 618 (5.8) |
| Chile | 268 (3.8) | 332 (3.2) | 380 (3.3) | 435 (4.7) | 523 (5.2) |
| Chinese Taipei | 411 (6.0) | 520 (5.8) | 595 (4.5) | 656 (4.5) | 731 (4.8) |
| Cyprus | 347 (5.5) | 418 (2.5) | 469 (2.3) | 515 (2.0) | 574 (2.7) |
| Egypt | 263 (8.8) | 346 (4.6) | 409 (3.5) | 475 (4.5) | 564 (4.4) |
| England | 390 (6.8) | 445 (5.8) | 486 (4.9) | 532 (7.3) | 593 (6.9) |
| Estonia | 434 (5.5) | 495 (4.7) | 538 (3.4) | 581 (4.1) | 641 (5.7) |
| Ghana | 43 (5.2) | 152 (4.9) | 229 (5.8) | 310 (8.9) | 427 (8.7) |
| Hong Kong, SAR | 467 (9.9) | 549 (5.3) | 594 (4.3) | 636 (3.2) | 689 (4.0) |
| Hungary | 416 (4.8) | 487 (4.5) | 538 (3.3) | 587 (2.7) | 654 (4.9) |
| Indonesia | 284 (8.2) | 364 (6.3) | 421 (4.8) | 480 (4.9) | 560 (4.3) |
| Iran, Islamic Rep. of | 286 (2.9) | 352 (2.1) | 401 (2.7) | 456 (3.4) | 533 (3.0) |
| Israel | 374 (4.8) | 450 (4.2) | 503 (3.8) | 553 (3.1) | 622 (6.0) |
| Italy | 358 (5.0) | 435 (3.6) | 487 (3.0) | 535 (3.3) | 602 (7.5) |
| Japan | 443 (3.2) | 519 (3.8) | 567 (2.1) | 611 (1.7) | 677 (6.6) |
| Jordan | 273 (6.0) | 364 (5.1) | 429 (6.0) | 494 (4.9) | 580 (8.7) |
| Korea, Rep. of | 444 (7.0) | 539 (3.9) | 599 (2.8) | 650 (2.7) | 717 (4.4) |
| Latvia | 408 (4.9) | 474 (4.4) | 520 (3.2) | 564 (3.0) | 625 (5.2) |
| Lebanon | 333 (6.0) | 398 (3.8) | 447 (3.9) | 496 (4.5) | 563 (4.2) |
| Lithuania | 390 (6.4) | 462 (3.3) | 513 (3.7) | 561 (3.2) | 627 (4.0) |
| Macedonia, Rep. of | 298 (8.8) | 388 (4.3) | 450 (5.6) | 508 (3.6) | 584 (5.8) |
| Malaysia | 393 (5.8) | 456 (3.8) | 505 (4.8) | 556 (5.1) | 619 (5.0) |
| Moldova, Rep. of | 325 (5.7) | 411 (5.6) | 471 (4.3) | 525 (6.0) | 593 (4.9) |
| Morocco | 260 (8.1) | 334 (4.9) | 386 (4.9) | 438 (4.5) | 512 (8.4) |
| Netherlands | 422 (3.8) | 480 (4.3) | 522 (4.3) | 563 (3.3) | 611 (5.4) |
| New Zealand | 377 (7.7) | 440 (3.7) | 484 (4.7) | 529 (7.9) | 596 (14.3) |
| Norway | 352 (3.4) | 413 (2.2) | 453 (2.5) | 490 (2.5) | 538 (2.3) |
| Palestinian Nat'l Auth. | 228 (5.1) | 320 (4.0) | 390 (4.6) | 462 (4.5) | 554 (5.1) |
| Philippines | 249 (4.7) | 326 (3.6) | 383 (6.0) | 448 (6.9) | 538 (7.4) |
| Romania | 327 (8.6) | 421 (6.4) | 491 (7.1) | 553 (5.3) | 629 (7.0) |
| Russian Federation | 395 (7.4) | 469 (3.5) | 520 (3.2) | 570 (3.8) | 640 (5.5) |
| Saudi Arabia | 170 (8.7) | 256 (5.8) | 312 (5.7) | 373 (4.9) | 465 (9.0) |
| Scotland | 377 (4.2) | 441 (4.0) | 483 (3.5) | 524 (3.4) | 575 (5.6) |
| Serbia | 357 (4.4) | 441 (3.1) | 499 (4.3) | 554 (2.8) | 620 (3.5) |
| Singapore | 461 (5.3) | 546 (4.8) | 598 (3.9) | 642 (1.9) | 696 (3.2) |
| Slovak Republic | 388 (8.3) | 467 (4.0) | 519 (4.4) | 569 (3.5) | 637 (4.7) |
| Slovenia | 391 (5.4) | 455 (2.0) | 499 (2.4) | 543 (2.1) | 606 (7.3) |
| South Africa | 116 (4.1) | 190 (3.2) | 247 (4.3) | 313 (7.1) | 477 (22.1) |
| Sweden | 392 (4.0) | 450 (3.7) | 487 (2.2) | 523 (1.5) | 575 (3.1) |
| Tunisia | 290 (4.7) | 351 (2.8) | 396 (3.1) | 444 (3.2) | 517 (6.4) |
| United States | 397 (3.2) | 462 (3.6) | 509 (3.2) | 557 (3.3) | 623 (3.9) |
| Benchmarking Participants |  |  |  |  |  |
| Basque Country, Spain | 395 (8.4) | 456 (3.2) | 497 (2.2) | 536 (1.8) | 589 (4.6) |
| Indiana State, US | 415 (6.8) | 473 (4.1) | 514 (5.1) | 556 (6.6) | 618 (9.6) |
| Ontario Province, Can. | 419 (5.4) | 473 (3.7) | 513 (2.4) | 552 (3.0) | 605 (4.2) |
| Quebec Province, Can. | 452 (2.9) | 500 (2.7) | 537 (3.1) | 574 (4.4) | 624 (4.4) |

## Exhibit D.1: Percentiles of Achievement in Knowing Cognitive Domain

| Countries | 5th Percentile | 25th <br> Percentile | 50th Percentile | 75th Percentile | 95th <br> Percentile |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Armenia | 296 (7.6) | 387 (5.9) | 447 (5.4) | 507 (5.0) | 597 (4.6) |
| Australia | 363 (8.0) | 449 (5.7) | 505 (4.6) | 554 (3.3) | 631 (5.7) |
| Belgium (Flemish) | 450 (3.4) | 512 (2.3) | 558 (2.3) | 604 (3.7) | 665 (2.9) |
| Chinese Taipei | 443 (4.5) | 518 (2.6) | 566 (2.2) | 614 (3.1) | 681 (4.1) |
| Cyprus | 351 (3.9) | 443 (4.6) | 503 (4.3) | 560 (2.4) | 641 (5.6) |
| England | 378 (7.7) | 472 (4.4) | 536 (5.1) | 599 (5.0) | 687 (4.5) |
| Hong Kong, SAR | 458 (4.5) | 528 (3.6) | 576 (3.6) | 620 (4.0) | 683 (3.7) |
| Hungary | 382 (4.2) | 464 (3.2) | 520 (3.5) | 572 (4.8) | 644 (5.7) |
| Iran, Islamic Rep. of | 275 (8.3) | 351 (3.2) | 403 (3.3) | 457 (5.0) | 533 (3.1) |
| Italy | 365 (5.4) | 456 (3.0) | 514 (3.8) | 573 (4.2) | 660 (6.5) |
| Japan | 424 (4.2) | 512 (3.7) | 566 (3.0) | 620 (2.7) | 696 (4.5) |
| Latvia | 403 (6.6) | 473 (4.0) | 519 (2.8) | 562 (2.9) | 623 (5.2) |
| Lithuania | 392 (5.0) | 469 (3.6) | 522 (2.9) | 571 (3.1) | 642 (4.7) |
| Moldova, Rep. of | 337 (11.3) | 437 (5.0) | 504 (5.5) | 566 (6.0) | 654 (9.3) |
| Morocco | 217 (5.2) | 299 (6.2) | 360 (4.6) | 420 (5.0) | 500 (7.4) |
| Netherlands | 440 (3.5) | 494 (2.9) | 531 (2.4) | 566 (1.9) | 617 (3.1) |
| New Zealand | 349 (5.1) | 437 (3.5) | 494 (3.0) | 551 (2.7) | 629 (4.3) |
| Norway | 315 (4.6) | 398 (2.8) | 451 (2.6) | 501 (3.2) | 568 (4.2) |
| Philippines | 231 (6.6) | 315 (4.5) | 378 (3.9) | 449 (8.9) | 559 (14.3) |
| Russian Federation | 381 (5.9) | 455 (5.8) | 511 (6.7) | 570 (5.1) | 655 (8.1) |
| Scotland | 356 (3.0) | 434 (4.1) | 485 (3.5) | 535 (5.6) | 605 (5.7) |
| Singapore | 442 (12.5) | 563 (9.2) | 633 (7.4) | 697 (7.2) | 784 (6.7) |
| Slovenia | 351 (5.4) | 424 (3.4) | 473 (2.3) | 517 (3.8) | 580 (3.3) |
| Tunisia | 188 (7.8) | 275 (5.3) | 338 (6.0) | 400 (5.6) | 487 (3.5) |
| United States | 396 (4.1) | 474 (2.7) | 529 (2.3) | 584 (3.5) | 657 (3.7) |
| Benchmarking Participants |  |  |  |  |  |
| Indiana State, US | 425 (5.8) | 497 (6.0) | 544 (4.5) | 591 (3.8) | 661 (7.9) |
| Ontario Province, Can. | 392 (5.3) | 465 (4.2) | 513 (3.3) | 562 (4.4) | 635 (8.7) |
| Quebec Province, Can. | 392 (5.0) | 459 (2.9) | 504 (3.6) | 551 (3.5) | 613 (7.6) |

[^10]| Countries | 5th Percentile | $\begin{aligned} & \text { 25th } \\ & \text { Percentile } \end{aligned}$ | 50th Percentile | 75th <br> Percentile | 95th Percentile |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Armenia | 326 (4.5) | 421 (3.8) | 483 (4.1) | 541 (3.7) | 607 (2.2) |
| Australia | 365 (7.6) | 452 (5.4) | 510 (5.4) | 567 (6.2) | 645 (6.8) |
| Bahrain | 272 (2.0) | 345 (2.9) | 398 (2.6) | 451 (2.5) | 522 (3.7) |
| Belgium (Flemish) | 402 (6.2) | 494 (3.3) | 543 (3.0) | 587 (3.3) | 642 (3.6) |
| Botswana | 256 (4.9) | 320 (3.7) | 367 (3.0) | 416 (2.9) | 486 (6.3) |
| Bulgaria | 328 (6.4) | 412 (7.6) | 471 (3.7) | 531 (5.0) | 614 (7.0) |
| Chile | 261 (4.9) | 332 (3.5) | 386 (3.6) | 446 (5.5) | 537 (7.0) |
| Chinese Taipei | 400 (7.3) | 513 (4.8) | 594 (4.3) | 655 (4.3) | 733 (4.5) |
| Cyprus | 309 (3.5) | 396 (5.1) | 461 (2.8) | 521 (2.4) | 595 (3.0) |
| Egypt | 251 (5.9) | 336 (3.7) | 401 (5.0) | 470 (3.7) | 565 (5.9) |
| England | 377 (6.9) | 448 (6.1) | 500 (6.4) | 559 (7.0) | 636 (8.3) |
| Estonia | 412 (4.1) | 481 (2.7) | 527 (3.4) | 576 (2.3) | 646 (5.4) |
| Ghana | 162 (4.6) | 237 (4.9) | 292 (4.1) | 348 (6.6) | 430 (13.0) |
| Hong Kong, SAR | 448 (15.6) | 544 (4.6) | 591 (3.1) | 634 (3.1) | 690 (2.7) |
| Hungary | 389 (8.7) | 466 (4.2) | 524 (4.2) | 581 (4.0) | 654 (7.0) |
| Indonesia | 262 (6.0) | 344 (6.9) | 405 (5.7) | 471 (6.3) | 567 (7.1) |
| Iran, Islamic Rep. of | 296 (3.8) | 362 (2.7) | 413 (2.9) | 467 (3.5) | 546 (5.5) |
| Israel | 344 (6.4) | 433 (5.9) | 497 (4.1) | 558 (3.9) | 640 (5.3) |
| Italy | 349 (6.1) | 431 (3.1) | 486 (3.2) | 539 (3.2) | 610 (5.6) |
| Japan | 423 (3.2) | 512 (3.1) | 567 (2.5) | 618 (2.5) | 694 (7.5) |
| Jordan | 274 (5.6) | 360 (5.4) | 422 (5.6) | 484 (6.0) | 568 (6.2) |
| Korea, Rep. of | 434 (2.5) | 531 (2.3) | 591 (2.7) | 643 (2.6) | 711 (4.5) |
| Latvia | 376 (4.7) | 452 (3.5) | 506 (3.5) | 558 (5.8) | 626 (4.8) |
| Lebanon | 312 (3.6) | 376 (2.9) | 424 (4.0) | 475 (4.9) | 546 (4.4) |
| Lithuania | 361 (3.3) | 442 (3.9) | 499 (3.9) | 557 (3.9) | 635 (4.2) |
| Macedonia, Rep. of | 277 (5.8) | 368 (6.7) | 431 (3.9) | 491 (4.3) | 571 (12.6) |
| Malaysia | 386 (5.2) | 456 (4.1) | 511 (6.1) | 569 (4.6) | 638 (6.5) |
| Moldova, Rep. of | 321 (6.0) | 402 (3.9) | 462 (5.0) | 516 (3.2) | 582 (4.3) |
| Morocco | 278 (3.8) | 339 (2.5) | 383 (2.7) | 428 (3.4) | 495 (5.7) |
| Netherlands | 427 (7.1) | 495 (6.0) | 545 (4.9) | 593 (5.3) | 651 (4.4) |
| New Zealand | 365 (8.2) | 441 (6.8) | 497 (5.2) | 551 (6.5) | 630 (7.1) |
| Norway | 341 (6.6) | 420 (4.2) | 472 (2.6) | 521 (2.7) | 583 (4.4) |
| Palestinian Nat'l Auth. | 242 (5.3) | 325 (2.6) | 387 (2.5) | 453 (3.6) | 538 (3.1) |
| Philippines | 250 (5.1) | 320 (5.9) | 373 (4.9) | 433 (6.8) | 524 (11.3) |
| Romania | 321 (7.7) | 411 (4.5) | 477 (5.5) | 540 (6.6) | 621 (6.6) |
| Russian Federation | 371 (5.0) | 448 (4.7) | 504 (3.9) | 560 (3.5) | 634 (5.8) |
| Saudi Arabia | 216 (8.5) | 289 (5.2) | 338 (4.1) | 388 (4.7) | 463 (4.7) |
| Scotland | 373 (4.8) | 455 (6.5) | 509 (4.5) | 560 (5.8) | 624 (5.0) |
| Serbia | 304 (4.1) | 401 (4.7) | 468 (3.8) | 537 (4.2) | 622 (2.4) |
| Singapore | 461 (7.7) | 562 (5.7) | 621 (3.8) | 669 (2.8) | 725 (3.7) |
| Slovak Republic | 359 (6.6) | 443 (3.4) | 502 (5.4) | 562 (4.4) | 645 (4.2) |
| Slovenia | 370 (5.7) | 440 (2.5) | 491 (2.7) | 542 (2.9) | 614 (4.1) |
| South Africa | 125 (4.1) | 199 (3.2) | 254 (4.6) | 320 (7.4) | 486 (23.7) |
| Sweden | 374 (6.7) | 455 (4.7) | 507 (2.6) | 557 (3.4) | 627 (5.5) |
| Tunisia | 321 (2.4) | 376 (3.1) | 416 (2.5) | 459 (2.6) | 526 (5.0) |
| United States | 364 (4.2) | 444 (3.4) | 502 (4.4) | 560 (3.5) | 639 (4.6) |
| Benchmarking Participants |  |  |  |  |  |
| Basque Country, Spain | 370 (3.6) | 437 (2.9) | 483 (4.0) | 528 (2.8) | 588 (4.3) |
| Indiana State, US | 382 (10.9) | 456 (6.6) | 506 (5.5) | 559 (6.4) | 634 (9.8) |
| Ontario Province, Can. | 405 (3.2) | 474 (3.9) | 524 (3.5) | 572 (3.5) | 636 (3.1) |
| Quebec Province, Can. | 450 (2.1) | 504 (4.4) | 544 (4.1) | 585 (5.3) | 640 (7.3) |

[^11]
## Exhibit D.2: Percentiles of Achievement in Applying Cognitive Domain



[^12]| Countries | 5th Percentile | $\begin{aligned} & \text { 25th } \\ & \text { Percentile } \end{aligned}$ | 50th Percentile | $\begin{aligned} & \text { 75th } \\ & \text { Percentile } \end{aligned}$ | $\begin{aligned} & \text { 95th } \\ & \text { Percentile } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Armenia | 293 (5.9) | 402 (4.7) | 472 (5.2) | 541 (3.1) | 622 (2.6) |
| Australia | 375 (5.5) | 461 (5.0) | 520 (5.6) | 572 (5.5) | 642 (6.3) |
| Bahrain | 297 (2.8) | 374 (3.5) | 426 (2.7) | 477 (2.9) | 545 (5.9) |
| Belgium (Flemish) | 395 (5.5) | 487 (3.7) | 539 (2.3) | 586 (3.3) | 648 (3.3) |
| Botswana | 204 (3.6) | 292 (4.0) | 353 (4.0) | 415 (4.4) | 502 (3.9) |
| Bulgaria | 325 (6.8) | 412 (4.4) | 472 (5.2) | 533 (4.7) | 610 (5.4) |
| Chile | 261 (5.5) | 346 (5.0) | 408 (4.0) | 472 (4.3) | 561 (4.2) |
| Chinese Taipei | 414 (6.7) | 514 (5.0) | 581 (4.0) | 642 (3.2) | 721 (4.2) |
| Cyprus | 308 (3.2) | 399 (3.7) | 458 (2.6) | 516 (2.5) | 593 (6.3) |
| Egypt | 247 (6.8) | 335 (3.4) | 401 (3.4) | 464 (5.2) | 554 (5.6) |
| England | 373 (5.9) | 451 (6.6) | 509 (6.5) | 567 (6.5) | 643 (7.4) |
| Estonia | 399 (7.9) | 472 (3.6) | 524 (2.8) | 574 (3.6) | 643 (4.8) |
| Ghana | 149 (4.7) | 248 (4.0) | 314 (5.5) | 380 (5.9) | 471 (5.4) |
| Hong Kong, SAR | 436 (9.1) | 522 (4.0) | 574 (4.2) | 621 (2.6) | 684 (3.5) |
| Hungary | 402 (6.0) | 477 (3.6) | 530 (3.0) | 582 (4.7) | 655 (6.9) |
| Indonesia | 253 (7.5) | 344 (5.5) | 406 (6.1) | 468 (4.7) | 556 (7.6) |
| Iran, Islamic Rep. of | 295 (5.0) | 365 (3.2) | 417 (2.9) | 468 (2.7) | 540 (2.4) |
| Israel | 328 (4.1) | 419 (4.1) | 485 (5.4) | 547 (3.8) | 632 (5.4) |
| Italy | 360 (8.1) | 438 (4.1) | 491 (3.2) | 542 (3.3) | 610 (5.4) |
| Japan | 446 (7.1) | 528 (3.1) | 577 (1.5) | 625 (3.3) | 698 (5.1) |
| Jordan | 295 (6.1) | 377 (5.4) | 435 (4.1) | 490 (4.5) | 566 (6.6) |
| Korea, Rep. of | 441 (3.6) | 530 (2.3) | 585 (1.6) | 638 (2.3) | 712 (3.7) |
| Latvia | 367 (6.4) | 447 (4.2) | 501 (3.4) | 553 (3.7) | 629 (5.7) |
| Lebanon | 278 (6.9) | 355 (3.1) | 410 (3.4) | 464 (3.7) | 540 (4.4) |
| Lithuania | 354 (5.1) | 435 (4.3) | 490 (2.6) | 545 (2.7) | 618 (3.2) |
| Macedonia, Rep. of | 278 (8.3) | 376 (4.5) | 442 (4.9) | 504 (5.6) | 584 (4.6) |
| Malaysia | 386 (2.9) | 453 (2.8) | 504 (4.9) | 554 (4.1) | 618 (3.9) |
| Moldova, Rep. of | 309 (8.7) | 395 (6.2) | 456 (3.6) | 513 (3.6) | 590 (3.3) |
| Morocco | 259 (7.4) | 338 (4.2) | 392 (3.2) | 446 (4.5) | 520 (8.2) |
| Netherlands | 416 (6.2) | 490 (4.8) | 543 (4.0) | 594 (5.3) | 660 (8.3) |
| New Zealand | 374 (5.2) | 455 (5.5) | 513 (6.8) | 565 (8.1) | 639 (7.0) |
| Norway | 343 (6.0) | 427 (3.6) | 483 (3.0) | 534 (2.7) | 604 (3.3) |
| Palestinian Nat'l Auth. | 259 (6.1) | 346 (3.3) | 405 (3.9) | 465 (2.7) | 545 (5.6) |
| Philippines | 177 (6.1) | 282 (5.4) | 355 (7.4) | 433 (6.2) | 542 (8.0) |
| Romania | 297 (8.7) | 395 (4.6) | 461 (5.1) | 523 (5.5) | 611 (4.8) |
| Russian Federation | 365 (8.9) | 443 (4.7) | 497 (3.8) | 551 (4.4) | 623 (3.4) |
| Saudi Arabia | 205 (7.9) | 289 (6.1) | 349 (3.9) | 407 (3.7) | 489 (6.5) |
| Scotland | 375 (7.9) | 459 (4.0) | 518 (3.9) | 570 (4.1) | 638 (8.5) |
| Serbia | 323 (6.0) | 411 (3.4) | 471 (3.5) | 527 (2.7) | 604 (5.5) |
| Singapore | 424 (5.6) | 528 (4.8) | 591 (3.2) | 645 (2.9) | 717 (3.7) |
| Slovak Republic | 360 (5.5) | 450 (7.2) | 506 (4.9) | 560 (3.9) | 636 (4.4) |
| Slovenia | 372 (4.7) | 445 (2.8) | 495 (3.3) | 543 (1.9) | 610 (3.8) |
| South Africa | 111 (8.0) | 207 (3.6) | 277 (3.4) | 354 (7.1) | 505 (13.7) |
| Sweden | 364 (9.3) | 453 (4.0) | 510 (2.9) | 567 (3.3) | 642 (5.9) |
| Tunisia | 282 (4.9) | 351 (3.0) | 400 (3.3) | 447 (2.4) | 516 (7.7) |
| United States | 366 (5.0) | 448 (4.1) | 507 (4.8) | 564 (2.9) | 638 (4.5) |
| Benchmarking Participants |  |  |  |  |  |
| Basque Country, Spain | 371 (7.9) | 446 (3.1) | 496 (2.3) | 545 (1.9) | 610 (2.1) |
| Indiana State, US | 376 (10.4) | 451 (5.0) | 502 (4.7) | 555 (5.9) | 627 (8.7) |
| Ontario Province, Can. | 403 (9.6) | 479 (3.6) | 531 (3.8) | 577 (3.5) | 641 (4.8) |
| Quebec Province, Can. | 429 (6.3) | 494 (3.5) | 540 (2.8) | 583 (2.9) | 646 (11.0) |

[^13]
## Exhibit D.3: Percentiles of Achievement in Reasoning Cognitive Domain

| Countries | 5th Percentile | 25th Percentile | 50th Percentile | 75th <br> Percentile | 95th <br> Percentile |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Armenia | 305 (4.4) | 387 (3.9) | 445 (2.8) | 503 (3.9) | 588 (4.7) |
| Australia | 373 (5.5) | 455 (5.4) | 510 (4.9) | 561 (3.4) | 633 (5.5) |
| Belgium (Flemish) | 421 (4.3) | 494 (3.3) | 542 (3.0) | 591 (3.5) | 659 (4.3) |
| Chinese Taipei | 422 (7.8) | 513 (3.4) | 568 (2.9) | 618 (2.0) | 691 (5.9) |
| Cyprus | 369 (4.5) | 460 (3.2) | 519 (2.6) | 574 (3.3) | 653 (4.4) |
| England | 390 (8.1) | 479 (4.6) | 539 (3.3) | 598 (4.0) | 678 (4.9) |
| Hong Kong, SAR | 427 (6.4) | 514 (4.4) | 568 (4.4) | 618 (3.0) | 689 (3.9) |
| Hungary | 379 (5.6) | 467 (4.7) | 528 (2.9) | 584 (4.0) | 661 (4.3) |
| Iran, Islamic Rep. of | 267 (9.9) | 346 (4.5) | 400 (4.2) | 455 (3.4) | 531 (3.9) |
| Italy | 356 (4.7) | 441 (3.0) | 499 (3.4) | 556 (3.7) | 644 (6.0) |
| Japan | 423 (7.2) | 508 (1.6) | 563 (2.9) | 618 (3.5) | 694 (4.4) |
| Latvia | 393 (7.5) | 476 (2.7) | 533 (2.6) | 588 (3.6) | 664 (5.2) |
| Lithuania | 381 (4.9) | 469 (3.5) | 530 (3.5) | 586 (4.1) | 662 (5.9) |
| Moldova, Rep. of | 341 (9.7) | 436 (4.7) | 498 (6.1) | 556 (5.5) | 636 (5.9) |
| Morocco | 235 (5.7) | 314 (5.3) | 368 (4.5) | 423 (6.5) | 502 (6.2) |
| Netherlands | 422 (4.8) | 492 (3.2) | 536 (5.3) | 580 (4.2) | 643 (3.4) |
| New Zealand | 360 (3.7) | 448 (2.4) | 507 (3.4) | 561 (3.3) | 637 (5.3) |
| Norway | 322 (5.5) | 409 (2.4) | 471 (2.0) | 527 (5.1) | 608 (3.5) |
| Philippines | 201 (6.9) | 287 (4.5) | 352 (6.5) | 425 (10.1) | 544 (18.4) |
| Russian Federation | 386 (6.3) | 468 (5.1) | 526 (4.9) | 585 (5.0) | 665 (4.5) |
| Scotland | 366 (7.2) | 448 (4.3) | 500 (3.7) | 551 (4.1) | 623 (7.1) |
| Singapore | 403 (10.9) | 516 (7.7) | 579 (5.3) | 640 (8.3) | 722 (5.3) |
| Slovenia | 339 (8.7) | 429 (3.7) | 488 (3.2) | 543 (4.7) | 619 (4.3) |
| Tunisia | 179 (8.5) | 273 (3.8) | 339 (5.2) | 406 (4.8) | 504 (6.2) |
| United States | 388 (3.4) | 466 (3.8) | 520 (2.5) | 574 (2.6) | 647 (2.9) |
| Benchmarking Participants |  |  |  |  |  |
| Indiana State, US | 416 (4.1) | 484 (3.1) | 529 (2.8) | 573 (4.8) | 639 (9.0) |
| Ontario Province, Can. | 406 (4.4) | 476 (4.1) | 523 (3.8) | 571 (5.2) | 637 (8.0) |
| Quebec Province, Can. | 389 (4.9) | 464 (4.2) | 514 (2.3) | 562 (2.6) | 629 (3.9) |

[^14]| Countries | Overall |  | Girls |  | Boys |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | Standard <br> Deviation | Mean | Standard <br> Deviation | Mean | Standard Deviation |
| Armenia | 480 (2.9) | 77 (1.3) | 486 (3.2) | 74 (1.3) | 474 (3.4) | 79 (1.8) |
| Australia | 497 (4.0) | 69 (2.6) | 491 (5.1) | 66 (2.3) | 502 (5.2) | 72 (3.8) |
| Bahrain | 401 (2.3) | 83 (1.3) | 419 (2.8) | 77 (1.7) | 383 (3.2) | 85 (1.9) |
| Belgium (Flemish) | 537 (2.5) | 68 (2.2) | 534 (3.4) | 68 (3.0) | 541 (3.6) | 69 (2.5) |
| Botswana | 372 (2.8) | 73 (1.7) | 374 (3.0) | 72 (1.7) | 370 (3.6) | 75 (2.1) |
| Bulgaria | 486 (4.1) | 81 (2.4) | 487 (5.0) | 80 (3.0) | 484 (4.3) | 81 (2.9) |
| Chile | 386 (3.2) | 77 (1.6) | 378 (3.5) | 74 (1.7) | 393 (4.1) | 79 (2.2) |
| Chinese Taipei | 585 (4.5) | 98 (2.0) | 589 (5.0) | 92 (2.1) | 582 (5.0) | 103 (2.4) |
| Cyprus | 466 (2.0) | 70 (0.8) | 474 (2.1) | 67 (1.4) | 458 (2.6) | 71 (1.2) |
| Egypt | 411 (3.4) | 92 (1.5) | 413 (4.1) | 89 (1.5) | 409 (4.8) | 94 (2.3) |
| England | 489 (4.0) | 62 (2.7) | 488 (4.5) | 61 (2.6) | 489 (5.2) | 63 (3.5) |
| Estonia | 538 (2.7) | 63 (1.4) | 538 (3.2) | 63 (1.6) | 538 (3.0) | 63 (1.6) |
| Ghana | 232 (5.9) | 114 (2.7) | 220 (7.0) | 111 (3.1) | 243 (6.4) | 115 (3.0) |
| Hong Kong, SAR | 589 (3.3) | 67 (3.0) | 591 (3.7) | 66 (3.0) | 587 (4.6) | 69 (3.5) |
| Hungary | 536 (3.1) | 73 (1.7) | 537 (3.6) | 72 (2.0) | 536 (3.4) | 74 (2.1) |
| Indonesia | 422 (4.3) | 84 (2.2) | 423 (4.5) | 84 (2.8) | 421 (4.6) | 84 (2.2) |
| Iran, Islamic Rep. of | 405 (2.6) | 76 (1.2) | 412 (4.8) | 74 (1.8) | 401 (4.2) | 77 (2.5) |
| Israel | 501 (3.1) | 76 (1.7) | 499 (3.3) | 72 (1.7) | 503 (4.0) | 79 (2.3) |
| Italy | 484 (3.2) | 74 (2.2) | 483 (3.2) | 72 (2.5) | 485 (3.8) | 76 (2.4) |
| Japan | 564 (1.9) | 71 (1.3) | 564 (3.8) | 68 (3.7) | 564 (3.4) | 74 (2.3) |
| Jordan | 428 (4.7) | 94 (2.1) | 442 (5.5) | 91 (2.8) | 415 (6.2) | 94 (2.5) |
| Korea, Rep. of | 592 (2.1) | 83 (1.6) | 589 (2.9) | 81 (1.7) | 594 (2.4) | 85 (1.8) |
| Latvia | 518 (2.8) | 66 (1.4) | 523 (2.9) | 63 (1.6) | 514 (3.4) | 68 (1.9) |
| Lebanon | 447 (3.2) | 70 (1.5) | 444 (3.6) | 70 (1.6) | 452 (3.9) | 71 (2.1) |
| Lithuania | 511 (2.7) | 72 (1.4) | 514 (3.5) | 71 (1.8) | 507 (2.9) | 75 (1.7) |
| Macedonia, Rep. of | 447 (3.8) | 87 (2.4) | 452 (4.1) | 85 (2.5) | 442 (4.2) | 88 (2.8) |
| Malaysia | 506 (3.9) | 70 (2.1) | 511 (4.4) | 68 (2.5) | 501 (4.4) | 71 (2.4) |
| Moldova, Rep. of | 466 (4.1) | 81 (1.9) | 472 (4.9) | 80 (2.2) | 460 (4.6) | 82 (2.2) |
| Morocco | 386 (2.8) | 76 (1.5) | 383 (3.7) | 76 (2.1) | 392 (4.2) | 76 (2.6) |
| Netherlands | 520 (3.1) | 58 (2.4) | 518 (3.5) | 58 (2.5) | 522 (3.6) | 59 (2.6) |
| New Zealand | 485 (4.8) | 66 (3.0) | 484 (4.3) | 63 (2.8) | 486 (6.5) | 69 (3.6) |
| Norway | 450 (2.1) | 57 (1.3) | 451 (2.5) | 55 (1.6) | 449 (3.0) | 58 (1.6) |
| Palestinian Nat'l Auth. | 391 (3.7) | 100 (1.6) | 398 (4.8) | 98 (2.2) | 382 (5.6) | 100 (2.3) |
| Philippines | 388 (5.2) | 88 (2.5) | 395 (5.2) | 87 (2.5) | 379 (6.1) | 90 (3.4) |
| Romania | 485 (4.9) | 92 (1.9) | 489 (5.3) | 91 (2.0) | 481 (5.2) | 94 (2.4) |
| Russian Federation | 519 (3.4) | 74 (1.3) | 525 (3.5) | 72 (1.5) | 514 (3.7) | 77 (1.6) |
| Saudi Arabia | 315 (4.6) | 89 (2.1) | 309 (6.9) | 85 (4.0) | 319 (6.2) | 91 (2.5) |
| Scotland | 481 (3.2) | 60 (1.7) | 483 (3.9) | 58 (2.2) | 478 (3.4) | 61 (1.7) |
| Serbia | 495 (2.7) | 80 (1.6) | 502 (3.1) | 77 (1.9) | 489 (3.1) | 83 (1.7) |
| Singapore | 591 (3.1) | 71 (2.1) | 596 (2.9) | 68 (2.5) | 586 (3.7) | 74 (2.2) |
| Slovak Republic | 517 (3.3) | 75 (1.7) | 521 (3.4) | 72 (2.0) | 514 (3.9) | 78 (2.0) |
| Slovenia | 499 (2.2) | 65 (1.4) | 502 (2.6) | 62 (1.6) | 495 (2.9) | 68 (1.7) |
| South Africa | 261 (5.4) | 106 (5.3) | 260 (6.1) | 104 (6.1) | 261 (6.3) | 109 (6.0) |
| Sweden | 486 (2.1) | 55 (1.3) | 486 (2.5) | 54 (1.6) | 486 (2.3) | 56 (1.5) |
| Tunisia | 399 (3.0) | 69 (1.4) | 388 (3.6) | 69 (1.8) | 410 (3.0) | 66 (1.5) |
| United States | 510 (2.8) | 69 (1.4) | 508 (3.0) | 67 (1.5) | 512 (3.0) | 71 (1.6) |
| Benchmarking Participants |  |  |  |  |  |  |
| Basque Country, Spain | 495 (2.2) | 59 (1.2) | 498 (2.2) | 55 (1.1) | 492 (3.0) | 62 (2.0) |
| Indiana State, US | 515 (4.6) | 61 (2.7) | 512 (4.7) | 60 (2.7) | 518 (5.2) | 63 (2.9) |
| Ontario Province, Can. | 513 (2.6) | 57 (1.4) | 512 (2.9) | 57 (1.7) | 513 (3.1) | 57 (1.6) |
| Quebec Province, Can. | 537 (2.7) | 53 (1.5) | 535 (3.3) | 52 (1.7) | 540 (3.1) | 54 (1.8) |

[^15]
## Exhibit D.4: Standard Deviations of Achievement in Knowing Cognitive Domain

| Countries | Overall |  | Girls |  | Boys |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | Standard <br> Deviation | Mean | Standard <br> Deviation | Mean | Standard Deviation |
| Armenia | 447 (3.7) | 91 (2.2) | 455 (3.7) | 88 (2.1) | 439 (4.3) | 93 (2.7) |
| Australia | 501 (3.8) | 81 (2.2) | 501 (4.1) | 79 (2.3) | 502 (4.6) | 83 (3.0) |
| Belgium (Flemish) | 558 (2.1) | 66 (1.1) | 556 (2.6) | 65 (1.3) | 560 (2.9) | 68 (1.5) |
| Chinese Taipei | 565 (2.2) | 72 (1.4) | 564 (2.4) | 68 (1.8) | 566 (2.6) | 76 (1.6) |
| Cyprus | 500 (2.8) | 88 (1.3) | 496 (3.1) | 86 (1.8) | 504 (3.6) | 89 (1.5) |
| England | 534 (4.5) | 93 (2.1) | 534 (4.4) | 90 (2.4) | 534 (5.3) | 96 (2.5) |
| Hong Kong, SAR | 574 (3.3) | 69 (1.3) | 574 (3.9) | 65 (1.3) | 573 (3.6) | 72 (2.0) |
| Hungary | 517 (3.3) | 80 (2.0) | 516 (4.1) | 78 (2.6) | 518 (3.6) | 81 (2.1) |
| Iran, Islamic Rep. of | 404 (4.0) | 78 (2.0) | 411 (6.6) | 78 (3.3) | 399 (4.5) | 78 (1.9) |
| Italy | 514 (3.9) | 89 (2.1) | 510 (4.3) | 88 (2.8) | 518 (4.1) | 90 (2.0) |
| Japan | 564 (2.1) | 82 (1.1) | 565 (2.6) | 78 (1.4) | 564 (3.0) | 86 (1.4) |
| Latvia | 517 (2.9) | 67 (1.5) | 518 (3.1) | 64 (1.5) | 515 (3.5) | 70 (2.3) |
| Lithuania | 519 (2.7) | 75 (1.3) | 522 (3.3) | 74 (1.9) | 520 (3.6) | 77 (1.4) |
| Moldova, Rep. of | 500 (5.2) | 97 (3.2) | 507 (5.9) | 96 (3.9) | 495 (5.3) | 97 (3.2) |
| Morocco | 360 (4.4) | 87 (1.5) | 356 (5.6) | 88 (1.9) | 363 (4.3) | 86 (2.2) |
| Netherlands | 530 (2.2) | 54 (1.4) | 527 (3.0) | 54 (1.7) | 533 (2.6) | 54 (1.8) |
| New Zealand | 493 (2.2) | 84 (2.2) | 494 (2.9) | 83 (2.6) | 492 (2.5) | 85 (2.4) |
| Norway | 448 (2.1) | 77 (1.2) | 447 (2.5) | 75 (1.6) | 449 (3.1) | 80 (1.6) |
| Philippines | 385 (6.9) | 99 (5.3) | 389 (8.0) | 100 (6.1) | 380 (6.3) | 98 (4.6) |
| Russian Federation | 513 (5.3) | 83 (2.1) | 513 (5.8) | 83 (2.5) | 514 (5.8) | 84 (2.4) |
| Scotland | 484 (3.0) | 76 (1.8) | 478 (3.1) | 72 (2.2) | 489 (4.2) | 79 (1.9) |
| Singapore | 626 (6.5) | 104 (3.2) | 632 (6.4) | 98 (2.9) | 620 (7.2) | 108 (3.7) |
| Slovenia | 470 (2.6) | 70 (1.4) | 469 (3.1) | 67 (1.7) | 471 (3.3) | 72 (1.8) |
| Tunisia | 338 (4.2) | 91 (2.2) | 337 (4.8) | 90 (2.4) | 338 (4.6) | 91 (2.5) |
| United States | 528 (2.5) | 79 (1.3) | 525 (2.4) | 77 (1.4) | 532 (3.0) | 81 (1.6) |
| Benchmarking Participants |  |  |  |  |  |  |
| Indiana State, US | 544 (3.7) | 72 (1.9) | 541 (4.4) | 69 (2.2) | 546 (4.3) | 75 (2.3) |
| Ontario Province, Can. | 514 (4.4) | 73 (2.8) | 508 (4.4) | 70 (2.7) | 519 (5.7) | 76 (3.6) |
| Quebec Province, Can. | 504 (2.8) | 67 (1.5) | 501 (3.2) | 67 (1.9) | 508 (2.9) | 68 (1.6) |


| Countries | Overall |  | Girls |  | Boys |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | Standard Deviation | Mean | Standard <br> Deviation | Mean | Standard Deviation |
| Armenia | 478 (3.0) | 86 (1.5) | 482 (3.5) | 83 (1.8) | 473 (3.5) | 88 (2.0) |
| Australia | 508 (4.8) | 86 (3.0) | 501 (6.1) | 82 (3.0) | 516 (6.0) | 88 (4.2) |
| Bahrain | 398 (1.6) | 76 (1.2) | 411 (2.3) | 71 (1.1) | 384 (2.3) | 79 (1.7) |
| Belgium (Flemish) | 536 (2.7) | 72 (2.2) | 529 (3.3) | 71 (2.8) | 544 (3.7) | 73 (2.6) |
| Botswana | 369 (2.7) | 71 (1.7) | 370 (3.0) | 70 (2.0) | 368 (2.9) | 71 (1.7) |
| Bulgaria | 471 (4.7) | 87 (2.7) | 471 (6.0) | 87 (3.5) | 472 (4.9) | 87 (3.0) |
| Chile | 391 (3.3) | 84 (1.8) | 382 (3.6) | 81 (1.8) | 399 (4.2) | 86 (2.4) |
| Chinese Taipei | 582 (4.6) | 101 (1.8) | 584 (5.1) | 96 (1.9) | 580 (5.1) | 107 (2.4) |
| Cyprus | 457 (1.6) | 88 (1.3) | 465 (1.9) | 85 (1.5) | 450 (2.5) | 90 (1.8) |
| Egypt | 404 (3.4) | 96 (1.5) | 401 (4.3) | 93 (1.7) | 406 (4.9) | 98 (2.2) |
| England | 503 (4.8) | 79 (3.1) | 503 (5.4) | 78 (2.9) | 504 (6.0) | 80 (4.2) |
| Estonia | 528 (2.9) | 71 (1.4) | 531 (3.3) | 71 (1.7) | 526 (3.2) | 71 (1.8) |
| Ghana | 293 (4.0) | 82 (2.0) | 286 (4.9) | 79 (2.2) | 299 (4.8) | 83 (2.3) |
| Hong Kong, SAR | 584 (3.2) | 73 (3.1) | 584 (3.7) | 70 (3.0) | 584 (4.5) | 75 (3.8) |
| Hungary | 523 (3.4) | 82 (2.0) | 517 (3.8) | 80 (2.2) | 529 (4.0) | 83 (2.4) |
| Indonesia | 408 (4.9) | 93 (2.5) | 408 (5.0) | 92 (3.1) | 409 (5.3) | 93 (2.5) |
| Iran, Islamic Rep. of | 416 (2.5) | 77 (1.4) | 420 (4.6) | 74 (2.0) | 413 (4.1) | 78 (2.7) |
| Israel | 495 (3.6) | 91 (2.0) | 490 (3.7) | 87 (2.4) | 500 (4.6) | 94 (2.4) |
| Italy | 484 (3.2) | 79 (1.9) | 479 (3.0) | 76 (2.1) | 488 (4.0) | 82 (2.4) |
| Japan | 564 (2.2) | 82 (1.6) | 563 (4.4) | 78 (4.4) | 565 (3.6) | 85 (2.2) |
| Jordan | 422 (4.2) | 89 (2.0) | 436 (4.9) | 86 (2.7) | 409 (5.8) | 90 (2.2) |
| Korea, Rep. of | 584 (2.2) | 84 (1.4) | 581 (2.9) | 82 (1.6) | 587 (2.3) | 86 (1.6) |
| Latvia | 504 (3.4) | 76 (1.9) | 505 (3.5) | 74 (2.4) | 504 (4.1) | 79 (2.5) |
| Lebanon | 426 (3.3) | 71 (1.6) | 422 (3.7) | 70 (1.6) | 432 (4.2) | 72 (2.4) |
| Lithuania | 499 (2.8) | 84 (1.5) | 499 (3.2) | 82 (2.1) | 497 (3.3) | 87 (1.8) |
| Macedonia, Rep. of | 428 (3.8) | 89 (2.5) | 431 (4.2) | 87 (2.8) | 426 (4.3) | 91 (2.8) |
| Malaysia | 512 (4.4) | 78 (2.2) | 515 (5.1) | 76 (2.7) | 508 (4.8) | 79 (2.5) |
| Moldova, Rep. of | 457 (3.9) | 80 (2.0) | 462 (4.0) | 77 (2.4) | 453 (4.5) | 82 (2.3) |
| Morocco | 384 (2.9) | 66 (1.4) | 377 (3.4) | 66 (1.9) | 393 (3.3) | 65 (2.0) |
| Netherlands | 543 (3.7) | 69 (2.7) | 538 (4.0) | 68 (2.8) | 548 (4.3) | 69 (3.1) |
| New Zealand | 497 (5.3) | 80 (3.2) | 496 (4.7) | 77 (2.9) | 497 (7.2) | 83 (3.7) |
| Norway | 468 (2.7) | 74 (1.4) | 469 (2.8) | 72 (1.5) | 468 (3.4) | 76 (2.0) |
| Palestinian Nat'l Auth. | 388 (3.2) | 91 (1.6) | 389 (4.1) | 91 (2.0) | 388 (4.6) | 91 (2.2) |
| Philippines | 378 (4.8) | 83 (2.8) | 383 (4.8) | 82 (2.8) | 373 (5.5) | 85 (3.6) |
| Romania | 475 (5.0) | 91 (1.8) | 475 (5.4) | 90 (2.1) | 474 (5.3) | 93 (2.3) |
| Russian Federation | 503 (3.7) | 80 (1.6) | 503 (3.8) | 78 (1.8) | 503 (4.1) | 82 (2.2) |
| Saudi Arabia | 338 (3.6) | 75 (2.2) | 332 (6.1) | 72 (3.2) | 344 (4.5) | 76 (2.5) |
| Scotland | 505 (3.9) | 76 (2.2) | 506 (4.8) | 74 (2.8) | 504 (3.8) | 79 (2.3) |
| Serbia | 467 (2.9) | 97 (1.5) | 468 (3.5) | 94 (2.1) | 466 (3.1) | 100 (1.6) |
| Singapore | 611 (3.6) | 80 (2.5) | 617 (3.6) | 76 (2.8) | 606 (4.1) | 83 (2.6) |
| Slovak Republic | 502 (3.7) | 87 (1.8) | 499 (4.0) | 84 (2.0) | 505 (4.3) | 90 (2.2) |
| Slovenia | 491 (2.3) | 74 (1.1) | 491 (3.0) | 71 (1.6) | 491 (2.8) | 77 (1.5) |
| South Africa | 269 (5.3) | 106 (5.1) | 267 (5.9) | 103 (5.8) | 271 (6.5) | 109 (5.9) |
| Sweden | 505 (2.8) | 76 (1.6) | 504 (3.2) | 75 (1.8) | 506 (2.8) | 77 (2.0) |
| Tunisia | 419 (2.3) | 62 (1.3) | 407 (2.6) | 61 (1.7) | 433 (2.4) | 60 (1.3) |
| United States | 502 (3.4) | 83 (1.6) | 497 (3.5) | 81 (1.8) | 506 (3.5) | 85 (1.7) |
| Benchmarking Participants |  |  |  |  |  |  |
| Basque Country, Spain | 481 (2.3) | 66 (1.5) | 483 (2.5) | 62 (1.5) | 480 (3.2) | 70 (2.2) |
| Indiana State, US | 507 (5.9) | 76 (3.0) | 498 (5.7) | 73 (3.2) | 516 (6.7) | 78 (3.6) |
| Ontario Province, Can. | 522 (3.0) | 70 (1.1) | 520 (3.3) | 68 (1.5) | 525 (3.5) | 71 (1.5) |
| Quebec Province, Can. | 545 (3.0) | 58 (1.7) | 539 (3.6) | 57 (1.8) | 549 (3.4) | 59 (2.0) |

[^16]
## Exhibit D.5: Standard Deviations of Achievement in Applying Cognitive Domain

| Countries | Overall |  | Girls |  | Boys |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | Standard <br> Deviation | Mean | Standard <br> Deviation | Mean | Standard <br> Deviation |
| Armenia | 462 (3.2) | 82 (1.9) | 465 (3.2) | 80 (2.0) | 459 (3.7) | 84 (2.2) |
| Australia | 490 (3.8) | 83 (2.0) | 487 (4.3) | 81 (2.2) | 492 (4.4) | 86 (2.5) |
| Belgium (Flemish) | 546 (2.1) | 60 (1.1) | 544 (2.5) | 59 (1.3) | 548 (2.7) | 62 (1.4) |
| Chinese Taipei | 561 (1.9) | 60 (0.9) | 561 (2.0) | 56 (1.0) | 562 (2.2) | 64 (1.2) |
| Cyprus | 510 (2.8) | 86 (1.5) | 504 (3.1) | 83 (1.6) | 516 (2.9) | 88 (1.9) |
| England | 526 (4.1) | 93 (2.0) | 524 (4.1) | 90 (2.3) | 528 (4.9) | 96 (2.5) |
| Hong Kong, SAR | 577 (3.3) | 66 (1.2) | 576 (3.5) | 63 (1.4) | 577 (3.5) | 69 (1.7) |
| Hungary | 530 (3.4) | 79 (2.2) | 530 (4.0) | 78 (2.6) | 531 (3.7) | 79 (2.5) |
| Iran, Islamic Rep. of | 391 (3.8) | 78 (1.7) | 391 (6.1) | 77 (2.7) | 391 (4.8) | 80 (1.9) |
| Italy | 494 (3.6) | 83 (1.9) | 489 (4.3) | 82 (2.6) | 498 (3.5) | 84 (1.7) |
| Japan | 566 (2.1) | 81 (1.5) | 563 (2.6) | 77 (1.8) | 569 (2.3) | 85 (1.8) |
| Latvia | 545 (3.3) | 80 (1.7) | 545 (3.4) | 77 (2.1) | 546 (4.0) | 83 (2.7) |
| Lithuania | 542 (2.9) | 76 (1.5) | 541 (3.7) | 75 (2.3) | 545 (3.7) | 79 (1.7) |
| Moldova, Rep. of | 507 (4.8) | 90 (3.0) | 511 (5.3) | 89 (3.7) | 502 (5.0) | 90 (3.0) |
| Morocco | 349 (4.5) | 83 (1.8) | 345 (5.6) | 83 (2.2) | 352 (4.4) | 83 (1.9) |
| Netherlands | 541 (2.6) | 60 (1.3) | 538 (2.8) | 61 (2.0) | 545 (3.2) | 58 (1.8) |
| New Zealand | 486 (2.3) | 86 (1.9) | 485 (3.0) | 85 (2.5) | 486 (2.5) | 87 (1.8) |
| Norway | 446 (2.2) | 76 (1.5) | 443 (2.8) | 73 (1.5) | 449 (3.0) | 78 (2.1) |
| Philippines | 364 (7.5) | 101 (6.3) | 370 (8.8) | 101 (6.8) | 357 (6.7) | 100 (5.9) |
| Russian Federation | 542 (4.7) | 84 (2.0) | 539 (5.0) | 83 (2.3) | 545 (5.1) | 85 (2.5) |
| Scotland | 487 (3.5) | 80 (1.5) | 482 (3.6) | 76 (1.8) | 492 (4.6) | 84 (1.9) |
| Singapore | 595 (5.9) | 90 (3.3) | 599 (5.8) | 86 (3.0) | 590 (6.6) | 94 (3.8) |
| Slovenia | 477 (2.8) | 77 (1.6) | 474 (3.1) | 74 (1.9) | 481 (3.8) | 80 (2.3) |
| Tunisia | 348 (4.6) | 102 (2.3) | 351 (5.1) | 102 (2.5) | 346 (4.7) | 102 (2.6) |
| United States | 505 (2.6) | 77 (1.3) | 501 (2.8) | 75 (1.2) | 510 (2.9) | 80 (1.7) |
| Benchmarking Participants |  |  |  |  |  |  |
| Indiana State, US | 523 (3.3) | 72 (2.5) | 521 (3.4) | 68 (2.5) | 525 (4.2) | 76 (2.9) |
| Ontario Province, Can. | 498 (4.5) | 74 (3.4) | 491 (3.8) | 70 (2.5) | 505 (5.8) | 76 (4.4) |
| Quebec Province, Can. | 498 (2.7) | 66 (1.2) | 493 (3.2) | 65 (1.7) | 503 (2.9) | 67 (1.4) |


| Countries | Overall |  | Girls |  | Boys |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | Standard Deviation | Mean | Standard <br> Deviation | Mean | Standard Deviation |
| Armenia | 468 (2.8) | 100 (2.0) | 473 (3.4) | 98 (1.9) | 463 (4.3) | 103 (2.5) |
| Australia | 515 (4.0) | 82 (2.4) | 515 (5.1) | 80 (2.5) | 516 (5.1) | 84 (3.4) |
| Bahrain | 424 (2.2) | 76 (1.3) | 435 (2.5) | 71 (1.7) | 412 (3.2) | 79 (1.6) |
| Belgium (Flemish) | 533 (2.8) | 76 (2.2) | 531 (3.8) | 75 (2.8) | 536 (3.6) | 77 (2.6) |
| Botswana | 353 (3.7) | 91 (1.5) | 356 (3.5) | 90 (1.8) | 351 (4.5) | 92 (1.9) |
| Bulgaria | 471 (3.9) | 88 (2.5) | 471 (5.2) | 86 (2.9) | 471 (4.4) | 89 (2.9) |
| Chile | 409 (3.5) | 91 (1.6) | 406 (4.1) | 90 (2.0) | 412 (4.2) | 92 (1.9) |
| Chinese Taipei | 576 (4.2) | 93 (1.9) | 581 (4.3) | 88 (2.0) | 572 (4.8) | 97 (2.3) |
| Cyprus | 455 (1.7) | 87 (1.5) | 465 (2.3) | 83 (2.3) | 446 (2.4) | 90 (1.7) |
| Egypt | 400 (3.6) | 94 (1.5) | 402 (4.5) | 92 (1.7) | 399 (5.1) | 95 (2.1) |
| England | 509 (4.7) | 82 (2.9) | 513 (4.8) | 80 (2.5) | 506 (5.9) | 85 (3.8) |
| Estonia | 523 (3.0) | 75 (1.7) | 526 (3.4) | 75 (2.0) | 519 (3.4) | 74 (2.1) |
| Ghana | 313 (4.0) | 97 (1.5) | 309 (4.6) | 97 (2.4) | 317 (5.0) | 97 (1.8) |
| Hong Kong, SAR | 569 (3.1) | 76 (2.6) | 571 (3.5) | 73 (2.5) | 567 (4.4) | 78 (3.4) |
| Hungary | 529 (3.1) | 77 (1.7) | 530 (3.7) | 76 (2.1) | 528 (3.5) | 79 (2.1) |
| Indonesia | 406 (4.3) | 92 (2.3) | 405 (4.4) | 91 (3.0) | 406 (4.8) | 94 (2.2) |
| Iran, Islamic Rep. of | 417 (2.8) | 75 (1.3) | 423 (3.8) | 72 (1.7) | 413 (4.4) | 77 (2.1) |
| Israel | 483 (3.3) | 92 (1.8) | 483 (3.4) | 89 (2.6) | 483 (4.6) | 96 (2.9) |
| Italy | 489 (2.9) | 76 (1.7) | 486 (3.0) | 74 (1.9) | 491 (3.4) | 78 (2.1) |
| Japan | 576 (1.8) | 76 (1.3) | 575 (3.7) | 72 (3.0) | 576 (3.0) | 80 (2.0) |
| Jordan | 433 (3.7) | 83 (1.8) | 442 (4.1) | 81 (2.3) | 425 (5.3) | 83 (2.2) |
| Korea, Rep. of | 582 (1.7) | 82 (1.1) | 580 (2.4) | 79 (1.6) | 584 (2.1) | 85 (1.3) |
| Latvia | 500 (3.4) | 80 (1.8) | 504 (3.6) | 76 (1.7) | 496 (4.4) | 83 (2.6) |
| Lebanon | 410 (3.0) | 80 (2.1) | 407 (3.5) | 79 (2.1) | 413 (4.9) | 81 (2.8) |
| Lithuania | 489 (2.6) | 81 (1.3) | 492 (3.0) | 79 (1.6) | 484 (3.3) | 84 (2.1) |
| Macedonia, Rep. of | 438 (3.7) | 93 (2.3) | 444 (4.1) | 90 (2.8) | 432 (4.7) | 95 (2.5) |
| Malaysia | 503 (3.4) | 71 (1.5) | 505 (3.9) | 70 (1.9) | 501 (3.9) | 73 (1.7) |
| Moldova, Rep. of | 453 (4.0) | 86 (2.4) | 458 (4.2) | 85 (2.7) | 448 (4.6) | 86 (3.3) |
| Morocco | 391 (3.2) | 80 (2.0) | 387 (3.9) | 80 (2.0) | 397 (4.0) | 80 (2.8) |
| Netherlands | 541 (3.8) | 74 (2.5) | 540 (4.3) | 74 (2.5) | 542 (4.5) | 76 (2.8) |
| New Zealand | 509 (5.2) | 81 (2.5) | 519 (5.4) | 76 (2.9) | 499 (6.7) | 85 (2.9) |
| Norway | 479 (2.8) | 80 (1.7) | 486 (3.1) | 78 (1.9) | 472 (3.5) | 81 (2.1) |
| Palestinian Nat'l Auth. | 404 (2.7) | 87 (1.6) | 410 (3.8) | 84 (1.8) | 397 (4.2) | 89 (2.3) |
| Philippines | 358 (5.8) | 110 (2.4) | 363 (5.9) | 109 (2.5) | 350 (6.4) | 112 (3.4) |
| Romania | 458 (4.5) | 95 (2.0) | 460 (5.0) | 92 (2.8) | 456 (5.0) | 98 (2.3) |
| Russian Federation | 496 (3.6) | 79 (1.9) | 498 (4.0) | 77 (2.3) | 494 (3.8) | 81 (2.0) |
| Saudi Arabia | 348 (4.3) | 87 (2.7) | 347 (5.7) | 82 (2.8) | 349 (6.1) | 90 (3.9) |
| Scotland | 513 (3.4) | 81 (2.1) | 517 (4.3) | 79 (2.4) | 509 (3.4) | 82 (2.7) |
| Serbia | 468 (2.6) | 86 (1.0) | 472 (3.3) | 83 (1.5) | 464 (2.8) | 87 (1.3) |
| Singapore | 583 (3.5) | 88 (2.2) | 589 (3.3) | 84 (2.7) | 579 (4.4) | 92 (2.5) |
| Slovak Republic | 504 (3.2) | 83 (2.0) | 505 (3.3) | 80 (1.8) | 503 (4.2) | 87 (2.6) |
| Slovenia | 494 (2.5) | 72 (1.3) | 500 (3.1) | 68 (1.9) | 488 (3.2) | 76 (1.6) |
| South Africa | 287 (5.0) | 118 (4.0) | 287 (5.6) | 116 (4.6) | 286 (5.7) | 121 (4.9) |
| Sweden | 508 (3.3) | 84 (1.9) | 511 (4.1) | 83 (2.2) | 505 (3.3) | 86 (2.6) |
| Tunisia | 399 (2.7) | 71 (1.1) | 390 (3.3) | 71 (1.6) | 410 (3.3) | 69 (1.6) |
| United States | 505 (3.3) | 83 (1.5) | 505 (3.3) | 81 (1.7) | 506 (3.7) | 85 (1.8) |
| Benchmarking Participants |  |  |  |  |  |  |
| Basque Country, Spain | 494 (2.4) | 73 (1.5) | 500 (2.8) | 68 (1.6) | 488 (3.8) | 77 (2.4) |
| Indiana State, US | 503 (5.2) | 77 (3.0) | 502 (4.6) | 74 (2.6) | 503 (6.5) | 79 (3.7) |
| Ontario Province, Can. | 527 (3.0) | 72 (1.5) | 527 (3.8) | 71 (2.0) | 528 (3.6) | 73 (1.8) |
| Quebec Province, Can. | 539 (3.2) | 66 (1.7) | 537 (3.7) | 64 (2.2) | 540 (3.9) | 67 (1.6) |

[^17]
## Exhibit D.6: Standard Deviations of Achievement in Reasoning Cognitive Domain

| Countries | Overall |  | Girls |  | Boys |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | Standard <br> Deviation | Mean | Standard <br> Deviation | Mean | Standard Deviation |
| Armenia | 445 (3.1) | 86 (2.0) | 449 (3.4) | 84 (2.3) | 442 (3.4) | 87 (2.4) |
| Australia | 507 (3.6) | 79 (1.8) | 507 (3.9) | 77 (2.1) | 507 (4.2) | 80 (2.3) |
| Belgium (Flemish) | 541 (2.2) | 72 (1.5) | 541 (2.6) | 71 (2.0) | 541 (2.8) | 74 (1.7) |
| Chinese Taipei | 563 (2.2) | 81 (1.3) | 565 (2.6) | 76 (1.6) | 562 (2.7) | 85 (1.7) |
| Cyprus | 516 (2.4) | 86 (1.4) | 515 (2.7) | 83 (1.6) | 517 (3.0) | 88 (1.8) |
| England | 537 (3.5) | 87 (1.5) | 539 (4.0) | 85 (2.0) | 536 (4.2) | 90 (2.1) |
| Hong Kong, SAR | 564 (3.7) | 79 (1.4) | 565 (4.0) | 75 (1.7) | 563 (3.8) | 82 (2.0) |
| Hungary | 524 (3.2) | 86 (2.0) | 525 (4.0) | 85 (2.4) | 524 (3.8) | 87 (2.3) |
| Iran, Islamic Rep. of | 400 (3.4) | 80 (1.9) | 406 (6.0) | 79 (2.9) | 396 (4.3) | 81 (2.2) |
| Italy | 499 (4.0) | 88 (2.0) | 496 (4.7) | 86 (2.7) | 502 (4.1) | 89 (2.1) |
| Japan | 562 (1.7) | 83 (1.0) | 559 (2.1) | 79 (1.3) | 564 (2.6) | 86 (1.4) |
| Latvia | 531 (3.2) | 83 (1.7) | 531 (3.3) | 80 (2.0) | 531 (4.1) | 85 (2.6) |
| Lithuania | 526 (3.1) | 85 (1.8) | 527 (3.7) | 82 (2.5) | 529 (3.9) | 89 (2.0) |
| Moldova, Rep. of | 494 (4.9) | 90 (3.1) | 501 (5.5) | 90 (3.9) | 488 (5.6) | 89 (3.3) |
| Morocco | 368 (4.4) | 81 (2.1) | 366 (5.6) | 82 (3.1) | 370 (4.7) | 80 (2.1) |
| Netherlands | 535 (2.9) | 67 (1.4) | 533 (3.4) | 66 (1.6) | 536 (3.2) | 68 (1.9) |
| New Zealand | 503 (2.2) | 84 (1.7) | 502 (2.9) | 82 (2.2) | 504 (2.4) | 85 (1.9) |
| Norway | 468 (2.1) | 87 (1.8) | 466 (2.5) | 85 (1.7) | 470 (2.8) | 88 (2.8) |
| Philippines | 359 (7.4) | 104 (5.1) | 366 (8.8) | 106 (6.3) | 352 (6.6) | 101 (4.1) |
| Russian Federation | 526 (4.8) | 85 (1.9) | 524 (5.2) | 83 (2.0) | 528 (4.9) | 86 (2.6) |
| Scotland | 498 (3.1) | 78 (1.9) | 495 (3.5) | 75 (2.2) | 502 (4.0) | 81 (2.2) |
| Singapore | 574 (6.1) | 96 (3.5) | 578 (6.2) | 92 (3.4) | 570 (6.8) | 99 (4.0) |
| Slovenia | 485 (2.6) | 84 (1.9) | 486 (3.0) | 81 (2.6) | 485 (3.6) | 88 (2.4) |
| Tunisia | 340 (4.2) | 98 (2.2) | 340 (5.8) | 98 (2.7) | 339 (4.7) | 98 (2.3) |
| United States | 519 (2.5) | 78 (1.0) | 517 (2.6) | 76 (1.0) | 522 (2.9) | 80 (1.3) |
| Benchmarking Participants |  |  |  |  |  |  |
| Indiana State, US | 528 (3.4) | 67 (2.0) | 528 (3.9) | 64 (2.1) | 528 (4.1) | 70 (2.5) |
| Ontario Province, Can. | 523 (3.6) | 70 (2.2) | 520 (3.4) | 67 (1.7) | 525 (4.8) | 73 (3.0) |
| Quebec Province, Can. | 512 (2.6) | 73 (1.2) | 510 (3.0) | 72 (1.8) | 514 (3.2) | 73 (1.6) |


[^0]:    Represents years of schooling counting from the first year of ISCED Level 1.
    Met guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
    1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1).

[^1]:    Note: $5 \%$ of these comparisons would be statistically significant by chance alone.

[^2]:    Note: $5 \%$ of these comparisons would be statistically significant by chance alone.

[^3]:    Note: $5 \%$ of these comparisons would be statistically significant by chance alone.

[^4]:    Note: $5 \%$ of these comparisons would be statistically significant by chance alone.

[^5]:    Note: $5 \%$ of these comparisons would be statistically significant by chance alone.

[^6]:    - Met guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
    National Desired Population does not cover all of International Desired Population (see Exhibit C.1).
    2 National Defined Population covers less than $90 \%$ of National Desired Population (see Exhibit C.1).

[^7]:    Met guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).

    1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1).
    2 National Defined Population covers less than 90\% of National Desired Population (see Exhibit C.1).
    Nearly satisfied guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).

[^8]:    $\ddagger \quad$ Nearly satisfied guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
    \# Did not satisfy guidelines for sample participation rates (see Exhibit C.2).

[^9]:    Met guidelines for sample participation rates only after replacement schools were included (see Exhibit C.2).
    1 National Desired Population does not cover all of International Desired Population (see Exhibit C.1).

[^10]:    ( ) Standard errors appear in parentheses

[^11]:    () Standard errors appear in parentheses

[^12]:    () Standard errors appear in parentheses

[^13]:    () Standard errors appear in parentheses

[^14]:    () Standard errors appear in parentheses.

[^15]:    () Standard errors appear in parentheses

[^16]:    () Standard errors appear in parentheses

[^17]:    () Standard errors appear in parentheses

