

Chapter 3

PERFORMANCE ON ITEMS WITHIN EACH MATHEMATICS CONTENT AREA

This chapter presents five or six example items within each of the mathematics content areas, including the performance on each of the items for each of the TIMSS countries. The example items were selected to illustrate the different topics covered within each content area as well as the different performance expectations. The items also were chosen to show the range of item formats used within each area. To provide some sense of what types of items were answered correctly by higher-performing as compared to lower-performing students, the items show a range of difficulty within each content area. Finally, it should be noted that all these items and others are released for use by the public.¹

The presentation for each of the content areas begins with a brief description of the major topics included in the content area and a discussion of student performance in that content area. The discussion is followed by a table showing the percent correct on the example items for each of the TIMSS countries at both the seventh and eighth grades. After the table showing the country-by-country results, there is a figure relating achievement on each of the example items to performance on the TIMSS international mathematics scale. This “difficulty map” provides a pictorial representation of achievement on the scale in relation to achievement on the items. Following the difficulty map, each item is presented in its entirety. The correct answer is circled for multiple-choice items and shown in the answer space for short-answer items. For extended-response questions, the answer shown exemplifies the type of student responses that were given full credit. All of the responses shown have been reproduced from students’ actual test booklets.

WHAT HAVE STUDENTS LEARNED ABOUT FRACTIONS AND NUMBER SENSE?

The category of fractions and number sense included operations and problem solving with whole numbers, fractions, decimals, and percentages as well as estimating and rounding. Table 3.1 presents the percent of correct responses given by students in each of the TIMSS countries to each of the six example items presented within this category.

Figure 3.1 presents a pictorial representation of the relationship between performance on the TIMSS international mathematics scale and achievement on the six example items for fractions and number sense.² The international achievement on each example item is indicated both by the average percent correct across all countries at the seventh and eighth grades and by the international mathematics scale value, or

¹ The IEA retained about one-third of the TIMSS items as secure for possible future use in measuring international trends in mathematics and science achievement. All remaining items are available for general use.

² The three-digit item label shown in the lower right corner of the box locating each example item on the item difficulty map refers to the original item identification number used in the student test booklets.

Table 3.1**Percent Correct for Fractions and Number Sense Example Items - Lower and Upper Grades (Seventh and Eighth Grades*)**

Country	Example 1 Subtraction problem with whole numbers.		Example 2 Write a larger fraction.		Example 3 Distance on map.	
	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade
[†] Belgium (Fl)	96 (1.1)	93 (2.9)	82 (2.6)	81 (3.1)	84 (1.8)	84 (2.6)
[†] Belgium (Fr)	95 (1.4)	91 (1.6)	70 (2.9)	72 (2.6)	76 (2.7)	82 (3.1)
Canada	91 (1.6)	91 (1.7)	74 (2.4)	80 (1.6)	62 (2.9)	63 (2.0)
Cyprus	81 (1.9)	85 (2.2)	80 (2.4)	77 (2.4)	49 (2.9)	61 (2.7)
Czech Republic	97 (1.1)	97 (0.9)	81 (2.2)	83 (2.1)	76 (2.3)	83 (2.5)
^{†2} England	59 (3.2)	65 (3.2)	79 (3.1)	79 (2.6)	61 (3.4)	69 (3.1)
France	92 (1.5)	97 (1.2)	66 (1.8)	75 (2.4)	72 (2.6)	84 (2.0)
Hong Kong	90 (1.4)	89 (1.9)	86 (2.2)	85 (2.2)	59 (2.4)	64 (2.5)
Hungary	95 (1.3)	96 (1.2)	85 (2.0)	87 (1.9)	73 (2.4)	82 (2.0)
Iceland	91 (2.0)	89 (3.2)	82 (3.4)	89 (2.8)	69 (3.2)	68 (4.4)
Iran, Islamic Rep.	86 (2.4)	83 (2.6)	38 (4.0)	31 (3.2)	30 (3.0)	32 (3.2)
Ireland	93 (1.5)	94 (1.5)	83 (1.9)	82 (2.0)	58 (2.9)	67 (2.4)
Japan	89 (1.4)	93 (1.2)	85 (1.3)	87 (1.2)	76 (1.7)	79 (1.7)
Korea	91 (1.6)	89 (1.8)	77 (2.3)	84 (2.2)	65 (2.1)	74 (2.3)
¹ Latvia (LSS)	84 (2.3)	89 (2.1)	60 (2.6)	69 (3.1)	61 (2.8)	70 (2.8)
¹ Lithuania	88 (2.3)	92 (1.6)	61 (3.8)	67 (3.0)	50 (3.5)	67 (3.0)
New Zealand	69 (3.5)	71 (2.3)	81 (2.4)	80 (2.0)	64 (2.6)	67 (2.2)
Norway	85 (5.5)	87 (2.0)	73 (5.3)	84 (1.6)	68 (3.8)	65 (2.7)
Portugal	78 (2.4)	87 (1.7)	62 (2.4)	63 (2.7)	48 (2.8)	56 (2.6)
Russian Federation	92 (1.6)	92 (1.6)	78 (1.9)	83 (1.9)	66 (2.2)	77 (2.3)
[†] Scotland	75 (2.5)	72 (2.5)	76 (2.4)	81 (2.4)	55 (2.8)	65 (3.1)
Singapore	98 (0.6)	98 (0.7)	84 (2.1)	88 (1.6)	79 (2.4)	84 (1.6)
Slovak Republic	94 (1.0)	93 (1.3)	80 (1.9)	85 (1.8)	70 (2.3)	76 (2.3)
Spain	94 (1.5)	98 (0.7)	71 (2.2)	71 (2.0)	53 (2.7)	62 (2.3)
Sweden	84 (2.2)	88 (1.6)	74 (2.6)	78 (2.5)	76 (2.2)	77 (1.9)
¹ Switzerland	96 (0.9)	96 (1.1)	81 (2.0)	83 (2.0)	76 (2.5)	81 (2.5)
[†] United States	88 (2.1)	90 (1.1)	79 (2.2)	81 (1.9)	52 (3.4)	61 (2.5)
Countries Not Satisfying Guidelines for Sample Participation Rates (See Appendix A for Details):						
Australia	82 (2.4)	82 (1.7)	76 (2.3)	78 (1.6)	68 (2.7)	69 (1.8)
Austria	94 (1.3)	96 (1.2)	89 (2.0)	87 (1.7)	76 (2.5)	78 (3.6)
Bulgaria	84 (3.3)	78 (2.8)	65 (4.7)	64 (4.7)	66 (5.0)	75 (4.4)
Netherlands	88 (2.6)	82 (3.6)	86 (2.5)	76 (3.3)	71 (2.7)	74 (3.7)
Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details):						
Colombia	57 (3.5)	64 (4.0)	66 (3.5)	77 (2.8)	34 (3.1)	31 (3.1)
^{†1} Germany	93 (1.4)	89 (2.0)	80 (2.2)	81 (2.3)	68 (2.9)	72 (2.9)
Romania	80 (2.0)	79 (2.4)	61 (2.9)	64 (2.7)	50 (2.9)	50 (2.7)
Slovenia	95 (1.2)	98 (0.8)	77 (2.7)	77 (2.7)	71 (2.4)	76 (2.2)
Countries With Unapproved Sampling Procedures at Classroom Level (See Appendix A for Details):						
Denmark	86 (2.5)	88 (2.0)	64 (3.2)	65 (3.8)	73 (2.9)	85 (2.3)
Greece	87 (1.5)	91 (1.4)	82 (1.6)	77 (2.0)	42 (2.6)	50 (2.4)
[†] South Africa	57 (2.7)	56 (3.3)	45 (3.7)	50 (2.4)	23 (2.2)	24 (2.2)
Thailand	87 (1.6)	86 (1.6)	68 (2.3)	73 (2.1)	66 (2.4)	67 (2.2)
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):						
[†] Israel	—	95 (1.4)	—	80 (3.1)	—	59 (3.3)
Kuwait	—	52 (3.5)	—	37 (5.7)	—	30 (4.6)

*Seventh and eighth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (—) indicates data are not available. Israel and Kuwait did not test at the seventh grade.

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1994-95.

Table 3.1 (Continued)**Percent Correct for Fractions and Number Sense Example Items - Lower and Upper Grades (Seventh and Eighth Grades*)**

Country	Example 4 Actual weight from rounded value.		Example 5 Rate of fuel consumption.		Example 6 Percent increase in price.	
	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade
[†] Belgium (Fl)	65 (2.7)	65 (2.4)	37 (2.9)	49 (3.0)	37 (2.9)	33 (2.4)
[†] Belgium (Fr)	23 (2.1)	30 (2.6)	36 (2.8)	36 (2.6)	29 (3.1)	36 (4.4)
Canada	60 (1.8)	67 (1.7)	32 (2.0)	36 (2.0)	16 (1.3)	20 (1.7)
Cyprus	12 (1.2)	17 (1.9)	29 (2.8)	30 (2.5)	19 (2.4)	19 (2.8)
Czech Republic	69 (2.3)	80 (1.7)	43 (3.3)	43 (4.1)	29 (2.9)	38 (3.4)
^{†2} England	62 (2.5)	72 (2.5)	30 (2.7)	40 (2.9)	18 (2.4)	21 (2.5)
France	–	–	27 (2.4)	34 (2.5)	17 (2.3)	29 (2.7)
Hong Kong	47 (3.4)	56 (2.8)	44 (2.8)	48 (3.1)	47 (2.9)	54 (2.7)
Hungary	60 (2.0)	67 (2.0)	40 (2.3)	46 (3.0)	36 (2.3)	46 (2.8)
Iceland	51 (2.6)	59 (4.1)	39 (4.0)	25 (4.1)	9 (1.9)	24 (3.2)
Iran, Islamic Rep.	5 (1.6)	6 (1.1)	33 (2.5)	30 (2.3)	15 (2.9)	11 (2.2)
Ireland	65 (2.1)	68 (2.0)	44 (2.9)	42 (2.5)	35 (2.5)	39 (3.2)
Japan	67 (1.3)	76 (1.3)	–	–	34 (2.0)	41 (2.0)
Korea	80 (1.6)	85 (1.3)	41 (2.9)	50 (2.7)	36 (3.1)	37 (2.8)
¹ Latvia (LSS)	38 (2.0)	49 (2.5)	36 (3.0)	38 (3.3)	14 (2.4)	17 (2.4)
[†] Lithuania	37 (2.5)	47 (2.5)	36 (2.9)	38 (3.3)	12 (2.0)	14 (2.5)
New Zealand	65 (2.0)	74 (1.8)	36 (2.7)	40 (2.7)	21 (2.3)	30 (2.4)
Norway	64 (2.4)	77 (1.6)	37 (3.6)	37 (2.7)	16 (2.6)	29 (2.5)
Portugal	29 (1.9)	33 (1.9)	32 (2.3)	37 (2.6)	10 (1.4)	11 (1.6)
Russian Federation	54 (2.0)	59 (2.8)	42 (2.5)	41 (2.9)	16 (1.8)	26 (2.4)
[†] Scotland	62 (2.6)	74 (2.0)	32 (2.5)	38 (2.9)	19 (2.2)	25 (3.2)
Singapore	82 (2.2)	89 (1.3)	62 (3.1)	70 (2.6)	69 (3.0)	78 (2.4)
Slovak Republic	41 (2.0)	52 (2.1)	33 (2.3)	38 (2.4)	20 (2.3)	34 (2.6)
Spain	17 (1.4)	28 (2.1)	30 (2.5)	25 (2.2)	11 (1.6)	11 (1.6)
Sweden	80 (1.7)	88 (1.3)	34 (2.8)	43 (2.8)	19 (2.3)	32 (2.1)
[†] Switzerland	49 (2.0)	59 (1.8)	34 (2.1)	44 (2.1)	16 (2.1)	25 (1.8)
[†] United States	57 (2.1)	66 (2.1)	32 (2.1)	34 (1.8)	14 (2.1)	20 (1.8)
Countries Not Satisfying Guidelines for Sample Participation Rates (See Appendix A for Details):						
Australia	73 (1.7)	81 (1.4)	34 (2.5)	42 (2.2)	21 (2.0)	28 (1.9)
Austria	57 (2.4)	63 (2.1)	31 (2.3)	33 (2.7)	32 (2.9)	40 (2.7)
Bulgaria	32 (3.3)	44 (3.8)	41 (5.2)	63 (5.2)	24 (3.3)	29 (4.6)
Netherlands	51 (2.1)	61 (2.9)	32 (3.1)	50 (3.5)	33 (3.7)	44 (3.1)
Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details):						
Colombia	6 (0.9)	6 (1.1)	33 (4.5)	29 (3.4)	11 (2.1)	11 (2.0)
^{††} Germany	48 (2.5)	55 (2.4)	37 (3.1)	37 (2.7)	27 (2.8)	32 (3.5)
Romania	25 (1.9)	26 (2.0)	33 (2.4)	39 (2.9)	13 (1.9)	20 (2.2)
Slovenia	27 (1.8)	38 (2.4)	32 (2.4)	31 (2.9)	21 (2.4)	31 (2.6)
Countries With Unapproved Sampling Procedures at Classroom Level (See Appendix A for Details):						
Denmark	59 (2.7)	71 (2.0)	30 (2.7)	31 (3.5)	17 (3.2)	22 (2.3)
Greece	49 (2.0)	56 (2.0)	29 (2.1)	29 (2.6)	20 (2.0)	19 (2.0)
[†] South Africa	20 (2.0)	16 (2.2)	24 (2.1)	23 (2.1)	24 (1.7)	18 (1.7)
Thailand	40 (2.4)	40 (2.4)	38 (2.8)	44 (2.7)	26 (2.3)	33 (3.2)
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):						
[†] Israel	–	63 (3.6)	–	41 (5.1)	–	31 (4.5)
Kuwait	–	10 (1.6)	–	22 (2.3)	–	13 (2.6)

*Seventh and eighth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

^{††}National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

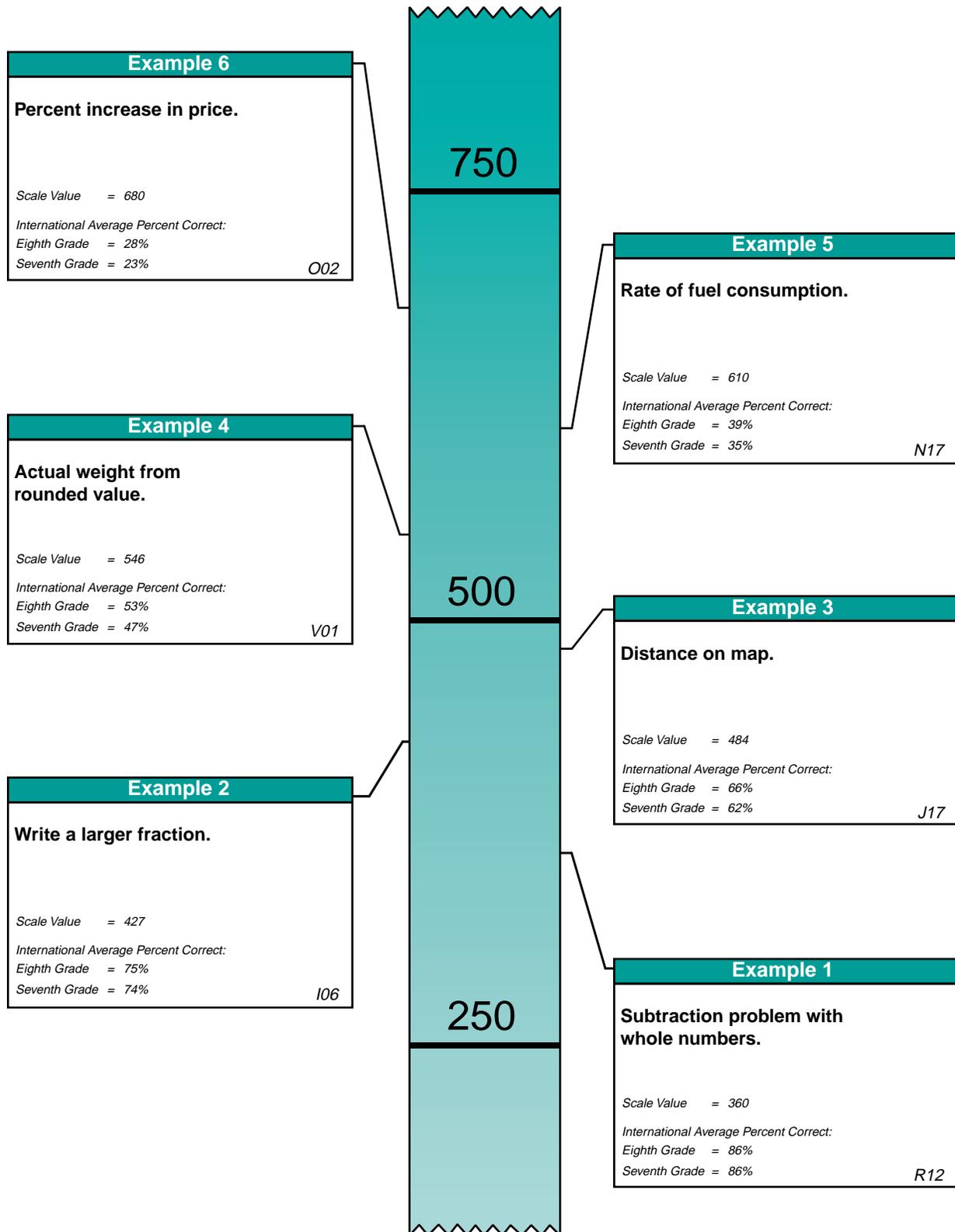
²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (–) indicates data are not available. Israel and Kuwait did not test at the seventh grade. Internationally comparable data are unavailable for France on Example 4 and Japan on Example 5.

Figure 3.1

**International Difficulty Map for Fractions and Number Sense Example Items
Lower and Upper Grades (Seventh and Eighth Grades*)**



*Seventh and eighth grades in most countries; see Table 2 for information about the grades tested in each country.

NOTE: Each item was placed onto the TIMSS international mathematics scale based on students' performance in both grades. Items are shown at the point on the scale where students with that level of proficiency had a 65 percent probability of providing a correct response.

item difficulty level, for each item. Since the scale was developed based on the performance of students at both grades in all countries, the international scale values apply to both grades and to all countries.

For the figure, the item results have been placed on the scale at the point where students at that level were more likely than not (65% probability) to answer the question correctly. For example, students scoring at or above 546 on the scale were likely to provide a correct response to the rounding item about the dolphin's actual weight (Example Item 4), and those scoring at or above 610 were likely to have responded correctly to the problem about rate of fuel consumption (Example Item 5). Considering that the international average on the scale was 513 at the eighth grade, however, students achieving at about the level of the international average were unlikely to have answered Example Item 5 (or Example Item 6 about percent increases) correctly. These results, however, varied dramatically by country. Eighth-grade students in Singapore, whose mean achievement was 643, had relatively high probabilities of answering all but the most difficult fractions and number sense items correctly. Indeed, this is borne out by Singapore's average percent correct of 79% in this content area at the eighth grade.

The six example items are presented in their entirety beginning on the next page. Example Item 1 is a subtraction problem with whole numbers that requires regrouping (borrowing). The international averages for the percent correct (86% for both grades) indicate that most seventh and eighth graders were successful on this item. In general, the lack of variation in performance between grades and across countries suggest that students in most countries have developed a grasp of how to solve this type of problem prior to the seventh and eighth grades.

Example Item 2 about understanding the relative size of fractions required students to provide their response, rather than select an answer in the multiple-choice format. On average, approximately three-fourths of both the seventh and eighth graders (74% and 75%, respectively) provided a correct response (any fraction larger than two-sevenths). Again, there were few differences in performance across countries or grade levels. With the exception of Iran, Kuwait, and South Africa, at least 60% of the seventh and eighth graders in each of the participating countries responded correctly.

Internationally, on average, about two-thirds of the students at seventh and eighth grades (62% and 66%) correctly interpreted the information about scale provided on the map shown in Example Item 3. As might be expected, the eighth graders performed better than seventh graders in many countries. Notwithstanding the between-grade increases, in all but a few cases, the majority of seventh graders answered the question correctly.

Averaged across countries, Example Item 4, which required students to demonstrate their understanding of rounded values, was answered correctly by approximately half the students at seventh and eighth grades (47% and 53%). Any value within the range of 165 through 174 was coded as a correct response. On this item, however, there was considerable variation in performance across countries. For example, 80% or more of the students at one or both grades in the Czech Republic, Korea, Singapore,

Sweden, and Australia provided a correct answer to this question. In contrast, fewer than 20% of the students did so at one or both grades in Cyprus, Iran, Spain, Colombia, Kuwait, and South Africa.

Multi-step problems such as the one shown in Example Item 5 were difficult for students around the world. On average, 35% of the seventh-grade students and 39% of those in eighth grade responded correctly. The most prevalent mistake was to select the amount of fuel used on the trip (option C) rather than the amount of fuel remaining in the tank.

The international averages for Example Item 6 indicate that working with percentages is a challenge for students in most countries. Only about one-fourth of the students at seventh and eighth grades (23% and 28%) responded correctly to this multiple-choice item. Singapore posted by far the best performance on this item (69% and 78% correct at grades 7 and 8), with Hong Kong having the next highest achievement (47% and 54% correct).

EXAMPLE ITEM 1 FRACTIONS & NUMBER SENSE

Subtraction problem with whole numbers

Subtract:

$$\begin{array}{r} 6000 \\ -2369 \\ \hline \end{array}$$

- A. 4369
- B. 3742
- C. 3631
- D. 3531

Performance Category: Performing Routine Procedures

EXAMPLE ITEM 2 FRACTIONS & NUMBER SENSE

Write a larger fraction

Write a fraction that is larger than $\frac{2}{7}$.

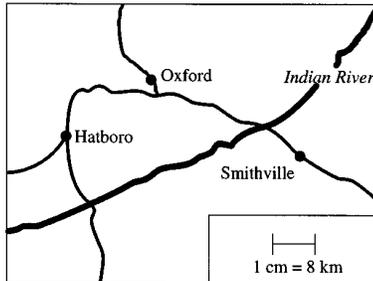
Answer: $\frac{3}{7}$

Performance Category: Knowing

EXAMPLE ITEM 3 FRACTIONS & NUMBER SENSE

Distance on map

One centimeter on the map represents 8 kilometers on the land.



About how far apart are Oxford and Smithville on the land?

- A. 4 km
- B. 16 km
- C. 35 km
- D. 50 km

Performance Category: Using Complex Procedures

EXAMPLE ITEM 4 FRACTIONS & NUMBER SENSE

Actual weight from rounded value

Rounded to the nearest 10 kg the weight of a dolphin was reported as 170 kg.
Write down a weight that might have been the actual weight of the dolphin.

Answer: 168

Performance Category: Using Complex Procedures

EXAMPLE ITEM 5
FRACTIONS & NUMBER SENSE**Rate of fuel consumption**

A car has a fuel tank that holds 35 L of fuel. The car consumes 7.5 L of fuel for each 100 km driven. A trip of 250 km was started with a full tank of fuel. How much fuel remained in the tank at the end of the trip?

- A. 16.25 L
- B. 17.65 L
- C. 18.75 L
- D. 23.75 L

Performance Category: Solving Problems

EXAMPLE ITEM 6
FRACTIONS & NUMBER SENSE**Percent increase in price**

If the price of a can of beans is raised from 60 cents to 75 cents, what is the percent increase in the price?

- A. 15%
- B. 20%
- C. 25%
- D. 30%

Performance Category: Performing Routine Procedures

WHAT HAVE STUDENTS LEARNED ABOUT GEOMETRY?

There was perhaps more variation in the geometry curriculum across countries than in any of the other mathematics content areas. The TIMSS geometry items required students to visualize geometric figures and to demonstrate their understanding of the properties of geometric figures. The concepts measured included symmetry, congruence, and similarity. Table 3.2 presents the results for the example items in geometry. Figure 3.2 presents the international difficulty map for the example items in geometry. Considering the international mean on the mathematics scale of 513 (for eighth grade), it can be seen that students performing above the mean were much more likely to understand the properties of geometric figures.

The range of student understanding in geometry is demonstrated by their performance on Example Items 7 through 12. Example Item 7 assessed spatial visualization skills, and Example Item 8 lines of symmetry. Although the content differed, internationally about two-thirds of the seventh- and eighth-grade students answered these questions correctly (Example Item 7 - 63% and 67%, Example Item 8 - 63% and 66%). Some countries did much better on these items than others. At the eighth grade, 80% or more students answered Example Item 7 correctly in Belgium (Flemish), the Czech Republic, Iceland, Japan, Latvia (LSS), the Slovak Republic, Switzerland, and Austria. This compares to fewer than half answering correctly in Cyprus, Iran, Colombia, South Africa, and Kuwait. Similarly, a number of countries were at about the 80% level on Example Item 8, while a few were at or below the level of 50% correct responses.

On average, Example Item 9, requiring understanding of ratio and perimeter, was answered correctly by 50% of the students at seventh grade and 56% at the eighth grade. In general, these international results reflect increases in achievement between the two grades shown in many countries and seem consistent with a curricular emphasis in geometry during the eighth grade.

The majority of students in many countries had difficulties with Example Item 10 on the properties of parallelograms. The international averages for the percents correct were 44% and 49% at the seventh and eighth grades, respectively. Only in Flemish-speaking Belgium (79%), Korea, (79%), and Bulgaria (78%) did more than three-fourths of the eighth-grade students answer this question correctly.

When given its coordinates and asked about another point on a line (Example Item 11), students showed great variation in performance from country to country. On average, the results were low at both seventh and eighth grades (38% and 41%). In the Netherlands, the top-performing country on this item, the corresponding figures were 62% and 66%. Students in England (58% and 55%) and Scotland (54% and 52%) also performed relatively well compared to their counterparts in other countries.

One of the most difficult geometry items assessed understanding of the properties of congruent triangles (Example Item 12). Internationally, the average percent of correct responses was 27% for the seventh grade and 35% for the eighth grade. Still, about two-thirds of the eighth-grade students responded correctly in Japan, Korea, and Singapore.

Table 3.2
**Percent Correct for Geometry Example Items
Lower and Upper Grades (Seventh and Eighth Grades*)**

Country	Example 7 Rotated 3-dimensional figure.		Example 8 Lines of symmetry.		Example 9 Ratio of side length to perimeter.	
	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade
[†] Belgium (Fl)	83 (1.8)	83 (2.1)	78 (2.2)	78 (3.3)	71 (2.7)	72 (3.5)
[†] Belgium (Fr)	76 (2.5)	74 (2.4)	71 (3.0)	80 (2.4)	66 (3.1)	62 (3.1)
Canada	68 (2.2)	75 (2.1)	78 (1.9)	76 (2.1)	51 (2.5)	69 (1.8)
Cyprus	49 (3.1)	43 (3.0)	56 (2.7)	58 (2.2)	35 (2.7)	55 (2.7)
Czech Republic	78 (1.9)	87 (1.9)	69 (2.8)	74 (2.6)	53 (2.6)	60 (2.9)
^{†2} England	72 (3.0)	77 (2.9)	79 (2.7)	82 (2.6)	49 (3.4)	52 (3.3)
France	71 (2.4)	77 (2.1)	79 (2.1)	80 (2.3)	58 (3.3)	69 (2.5)
Hong Kong	72 (3.0)	75 (2.7)	78 (2.6)	73 (2.4)	63 (3.6)	71 (2.6)
Hungary	61 (2.6)	71 (2.6)	80 (2.2)	82 (2.1)	43 (3.1)	55 (2.7)
Iceland	71 (3.1)	81 (2.2)	76 (2.4)	55 (3.5)	28 (2.7)	32 (3.1)
Iran, Islamic Rep.	52 (3.9)	42 (2.6)	68 (3.3)	68 (3.3)	57 (3.9)	50 (3.6)
Ireland	69 (2.2)	75 (2.5)	59 (2.6)	64 (2.6)	47 (2.6)	54 (3.2)
Japan	74 (1.9)	80 (1.3)	82 (1.6)	77 (1.6)	76 (1.8)	80 (1.6)
Korea	62 (2.5)	74 (2.6)	49 (3.0)	58 (2.7)	77 (2.0)	78 (2.1)
¹ Latvia (LSS)	85 (1.9)	81 (2.6)	45 (3.4)	50 (3.1)	40 (3.5)	54 (3.2)
¹ Lithuania	60 (3.0)	69 (3.1)	49 (3.2)	58 (3.6)	33 (2.8)	46 (3.0)
New Zealand	65 (2.9)	67 (2.3)	70 (2.7)	80 (2.0)	40 (2.6)	48 (2.5)
Norway	73 (2.9)	78 (2.1)	47 (3.1)	42 (2.7)	33 (3.0)	41 (2.5)
Portugal	51 (2.8)	58 (2.5)	46 (2.3)	44 (2.7)	45 (2.8)	48 (2.3)
Russian Federation	69 (2.4)	75 (2.8)	61 (2.4)	67 (3.3)	49 (3.1)	55 (4.3)
[†] Scotland	65 (2.6)	72 (2.3)	83 (2.3)	86 (1.7)	47 (2.8)	48 (3.0)
Singapore	77 (1.9)	79 (1.9)	77 (3.0)	81 (2.1)	75 (2.5)	80 (1.8)
Slovak Republic	71 (2.3)	81 (2.1)	70 (2.7)	75 (2.2)	59 (2.3)	67 (2.3)
Spain	68 (2.4)	71 (2.2)	47 (2.6)	51 (2.5)	48 (2.7)	55 (2.6)
Sweden	49 (3.0)	53 (2.6)	51 (2.7)	44 (2.4)	40 (2.8)	47 (2.5)
¹ Switzerland	79 (2.3)	82 (2.0)	58 (2.8)	76 (2.6)	44 (2.6)	55 (2.4)
[†] United States	63 (2.3)	62 (2.5)	66 (3.0)	70 (2.2)	45 (3.0)	55 (1.9)
Countries Not Satisfying Guidelines for Sample Participation Rates (See Appendix A for Details):						
Australia	69 (2.5)	73 (1.7)	70 (1.8)	69 (2.0)	54 (3.0)	60 (2.1)
Austria	70 (2.6)	80 (2.8)	53 (2.6)	57 (3.9)	54 (3.5)	69 (3.0)
Bulgaria	48 (3.5)	58 (5.3)	66 (4.3)	78 (4.7)	61 (5.2)	56 (3.4)
Netherlands	64 (3.3)	77 (2.7)	85 (2.4)	72 (3.9)	54 (2.7)	60 (4.5)
Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details):						
Colombia	46 (3.8)	41 (3.6)	40 (3.6)	44 (3.9)	30 (4.3)	37 (4.2)
^{†1} Germany	72 (2.2)	72 (2.7)	58 (3.1)	64 (3.1)	36 (3.2)	45 (3.3)
Romania	50 (2.8)	53 (2.4)	49 (2.5)	46 (2.7)	52 (2.9)	59 (2.8)
Slovenia	72 (2.3)	73 (2.5)	51 (2.8)	69 (2.5)	53 (2.4)	69 (2.7)
Countries With Unapproved Sampling Procedures at Classroom Level (See Appendix A for Details):						
Denmark	68 (3.4)	73 (3.1)	51 (3.2)	52 (3.2)	31 (3.5)	35 (3.1)
Greece	55 (2.1)	64 (2.7)	50 (2.4)	62 (3.0)	49 (2.3)	61 (2.2)
[†] South Africa	30 (2.2)	36 (2.3)	31 (2.6)	29 (2.3)	36 (2.3)	31 (2.5)
Thailand	42 (2.2)	50 (2.5)	79 (1.8)	80 (1.8)	56 (2.9)	64 (2.2)
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):						
[†] Israel	–	57 (3.5)	–	76 (3.5)	–	69 (3.5)
Kuwait	–	29 (3.1)	–	61 (4.2)	–	38 (4.8)

*Seventh and eighth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (–) indicates data are not available. Israel and Kuwait did not test at the seventh grade.

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1994-95.

Table 3.2 (Continued)**Percent Correct for Geometry Example Items
Lower and Upper Grades (Seventh and Eighth Grades*)**

Country	Example 10 Properties of parallelograms.		Example 11 Point on a line.		Example 12 Congruent triangles.	
	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade
[†] Belgium (Fl)	78 (2.5)	79 (2.0)	39 (2.4)	44 (3.5)	29 (2.8)	43 (2.8)
[†] Belgium (Fr)	50 (3.2)	57 (2.5)	24 (3.0)	23 (2.6)	29 (3.0)	32 (2.8)
Canada	48 (2.8)	48 (2.5)	43 (2.1)	49 (2.0)	20 (2.3)	29 (2.5)
Cyprus	37 (2.7)	41 (3.0)	29 (2.6)	30 (2.5)	33 (2.6)	41 (2.4)
Czech Republic	47 (3.0)	57 (3.0)	30 (2.9)	34 (3.1)	43 (3.7)	51 (3.0)
^{†2} England	39 (3.3)	48 (3.4)	58 (3.6)	55 (3.7)	24 (2.8)	31 (3.7)
France	48 (2.8)	62 (3.0)	24 (2.2)	34 (2.5)	38 (3.2)	50 (2.8)
Hong Kong	58 (3.4)	56 (2.5)	51 (2.5)	50 (2.8)	55 (3.0)	61 (2.7)
Hungary	42 (2.7)	57 (2.6)	47 (3.2)	51 (2.6)	28 (2.4)	39 (2.8)
Iceland	41 (4.7)	43 (3.3)	39 (4.2)	43 (3.4)	24 (3.2)	43 (3.6)
Iran, Islamic Rep.	30 (3.3)	31 (2.4)	22 (3.0)	17 (2.4)	28 (3.8)	35 (2.8)
Ireland	44 (2.5)	47 (2.9)	45 (2.7)	46 (2.6)	26 (2.2)	34 (2.6)
Japan	–	–	39 (2.1)	47 (2.2)	40 (2.1)	69 (1.7)
Korea	59 (2.3)	79 (2.1)	42 (3.0)	42 (3.2)	55 (2.8)	66 (2.1)
¹ Latvia (LSS)	27 (2.8)	51 (3.1)	34 (3.1)	38 (3.0)	20 (2.3)	25 (2.9)
¹ Lithuania	30 (3.5)	47 (3.2)	21 (3.0)	24 (2.8)	10 (2.0)	27 (2.8)
New Zealand	42 (2.7)	44 (2.8)	45 (3.1)	52 (2.8)	19 (2.0)	26 (2.5)
Norway	37 (3.6)	45 (2.6)	29 (3.2)	44 (3.1)	25 (2.5)	30 (2.3)
Portugal	33 (2.7)	33 (2.2)	35 (2.7)	46 (2.5)	21 (2.0)	21 (2.3)
Russian Federation	42 (2.4)	69 (3.3)	35 (3.3)	46 (3.3)	33 (3.2)	39 (2.9)
[†] Scotland	40 (3.1)	42 (2.5)	54 (2.7)	52 (3.1)	25 (2.2)	29 (2.7)
Singapore	58 (2.9)	57 (2.3)	47 (2.6)	59 (2.3)	55 (2.8)	69 (2.3)
Slovak Republic	43 (2.6)	46 (3.3)	33 (2.5)	40 (2.8)	35 (2.0)	45 (2.5)
Spain	39 (2.6)	40 (2.5)	37 (2.9)	39 (2.6)	17 (2.0)	14 (1.9)
Sweden	40 (2.7)	44 (2.6)	38 (2.5)	51 (2.3)	18 (2.3)	34 (2.4)
¹ Switzerland	39 (3.1)	52 (2.9)	46 (2.8)	51 (2.7)	25 (2.1)	33 (2.8)
[†] United States	39 (2.8)	40 (2.2)	37 (2.8)	41 (1.8)	15 (1.8)	17 (1.6)
Countries Not Satisfying Guidelines for Sample Participation Rates (See Appendix A for Details):						
Australia	44 (2.5)	46 (2.1)	47 (2.4)	51 (1.8)	29 (2.2)	34 (1.8)
Austria	49 (3.2)	48 (3.5)	46 (2.8)	54 (3.3)	32 (3.0)	29 (2.9)
Bulgaria	72 (4.0)	78 (4.5)	38 (4.5)	38 (5.1)	45 (5.4)	44 (5.1)
Netherlands	27 (2.9)	37 (3.8)	62 (3.4)	66 (4.5)	14 (2.4)	21 (3.0)
Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details):						
Colombia	32 (2.9)	34 (3.9)	24 (4.6)	28 (4.3)	8 (1.5)	12 (2.6)
^{†1} Germany	42 (3.1)	55 (3.2)	32 (2.9)	38 (2.9)	28 (2.7)	29 (3.0)
Romania	60 (2.9)	67 (2.9)	18 (2.0)	22 (2.3)	34 (2.5)	41 (2.9)
Slovenia	34 (2.9)	40 (2.9)	37 (2.8)	32 (2.9)	26 (2.7)	37 (3.3)
Countries With Unapproved Sampling Procedures at Classroom Level (See Appendix A for Details):						
Denmark	41 (3.4)	43 (3.0)	45 (3.0)	51 (3.7)	19 (2.7)	33 (3.2)
Greece	48 (2.7)	47 (2.7)	32 (2.2)	25 (2.4)	19 (2.2)	37 (2.3)
[†] South Africa	27 (2.2)	27 (2.0)	28 (2.2)	25 (2.2)	11 (1.3)	14 (1.8)
Thailand	62 (1.8)	62 (2.4)	47 (2.3)	44 (2.7)	22 (1.8)	33 (2.2)
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):						
¹ Israel	–	57 (3.1)	–	42 (3.6)	–	43 (3.4)
Kuwait	–	13 (2.4)	–	24 (3.0)	–	20 (3.2)

*Seventh and eighth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

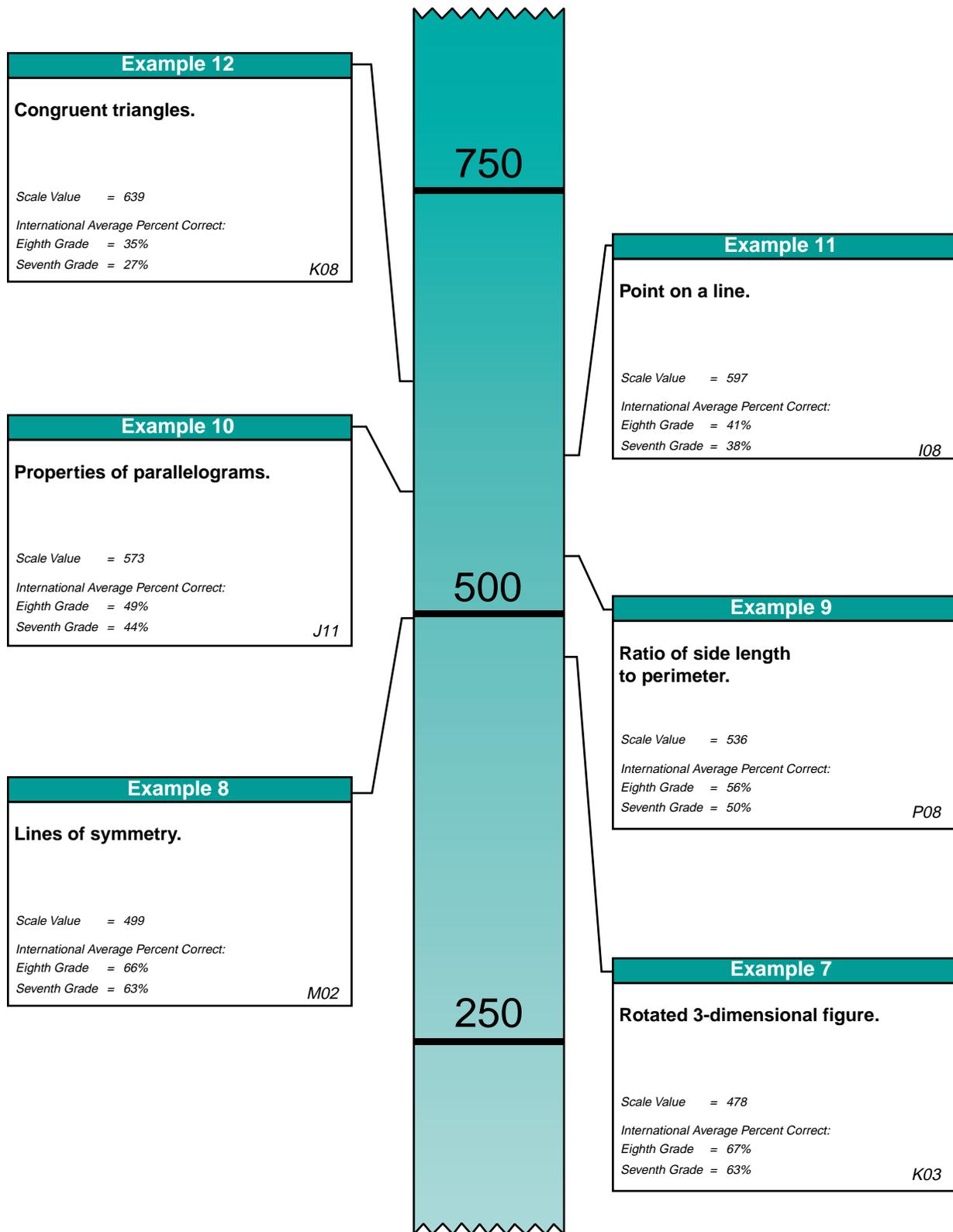
²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (–) indicates data are not available. Israel and Kuwait did not test at the seventh grade. Internationally comparable data are unavailable for Japan on Example 10.

Figure 3.2

**International Difficulty Map for Geometry Example Items
Lower and Upper Grades (Seventh and Eighth Grades*)**



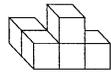
*Seventh and eighth grades in most countries; see Table 2 for information about the grades tested in each country.

NOTE: Each item was placed onto the TIMSS international mathematics scale based on students' performance in both grades. Items are shown at the point on the scale where students with that level of proficiency had a 65 percent probability of providing a correct response.

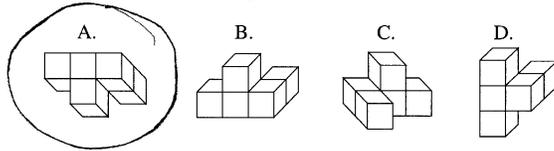
EXAMPLE ITEM 7
GEOMETRY

Rotated 3-dimensional figure

This figure will be turned to a different position.



Which of these could be the figure after it is turned?

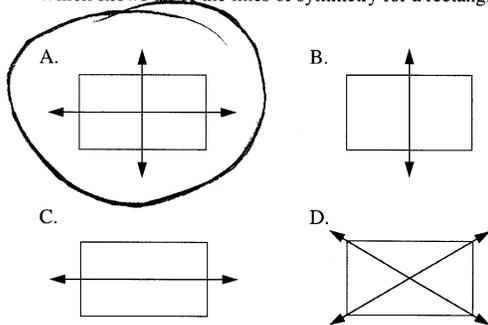


Performance Category: Using Complex Procedures

EXAMPLE ITEM 8
GEOMETRY

Lines of symmetry

Which shows all of the lines of symmetry for a rectangle?



Performance Category: Knowing

EXAMPLE ITEM 9
GEOMETRY**Ratio of side length to perimeter**

What is the ratio of the length of a side of a square to its perimeter?

A. $\frac{1}{1}$

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $\frac{1}{4}$

Performance Category: Solving Problems

EXAMPLE ITEM 10
GEOMETRY**Properties of parallelograms**

A quadrilateral **MUST** be a parallelogram if it has

A. one pair of adjacent sides equal

B. one pair of parallel sides

C. a diagonal as axis of symmetry

D. two adjacent angles equal

E. two pairs of parallel sides

Performance Category: Knowing

EXAMPLE ITEM 11

GEOMETRY

Point on a line

A straight line on a graph passes through the points (3,2) and (4,4). Which of these points also lies on the line?

- A. (1,1)
- B. (2,4)
- C. (5,6)
- D. (6,3)
- E. (6,5)

Performance Category: Solving Problems

EXAMPLE ITEM 12

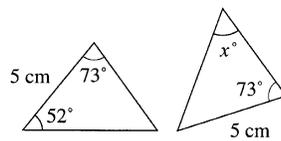
GEOMETRY

Congruent triangles

These triangles are congruent. The measures of some of the sides and angles of the triangles are shown.

What is the value of x ?

- A. 52
- B. 55
- C. 65
- D. 73
- E. 75



Performance Category: Performing Routine Procedures

WHAT HAVE STUDENTS LEARNED ABOUT ALGEBRA?

To demonstrate their understanding of algebraic concepts, students were asked to solve a variety of problems involving patterns, relations, expressions, and equations. The country-by-country results for the example algebra items are presented in Table 3.3. Figure 3.3, showing the relationship between performance on these items and performance on the mathematics scale, suggests that even some of the eighth graders in most countries had considerable difficulty with all but the most straightforward algebra questions. Questions involving expressions and equations were most likely to be answered correctly by only the higher-performing students (students achieving approximately at or above the eighth-grade mean of 513).

Example Items 13 through 17 illustrate the range of student performance. As shown by Example Item 13, the easiest items measured concepts underlying algebra such as the ability to detect patterns. In most countries, students performed very well on this item at both grades (87% and 90% correct responses averaged across countries).

Example Item 14 is a two-part item requiring students to supply their answers. In the first part of the item, students generally were able to establish the number of small triangles in the figures (72% and 75% average correct at the seventh and eighth grades, respectively). Of course, finding the answers of 4 and 9 could have been accomplished by actually counting the small triangles. In contrast, very few students demonstrated their ability to extend the pattern and determine that 64 small triangles would be needed for the 8th figure (international averages of 18% and 26%). In only Japan (52%) and Singapore (50%) did at least half the eighth-grade students provide a correct response to this question.

Example Items 15, 16, and 17 required students to work with algebraic equations and expressions. The international results for Example Item 15 indicate that students in most countries were relatively successful in solving a simple linear equation for x (on average, 62% and 72% correct at the seventh and eighth grades). As shown by the data for Example Item 16, they had more difficulty recognizing that $m + m + m + m$ was equivalent to $4m$ (international averages of 47% and 58%). It should be noted, however, that three-fourths or more of the eighth-grade students answered this question correctly in the Czech Republic, Hong Kong, Japan, the Russian Federation, Singapore, the Slovak Republic, and Slovenia. Considering the performance on Example Item 16, it is not surprising that students had even more difficulty identifying the correct expression to represent the number of Clarissa's hats as required by Example Item 17. International performance on this item averaged 37% at the seventh grade and 47% at the eighth grade.

Table 3.3**Percent Correct for Algebra Example Items
Lower and Upper Grades (Seventh and Eighth Grades*)**

Country	Example 13 Shapes in a pattern.		Example 14A Sequence of triangles: chart finding pattern.		Example 14B Sequence of triangles: extending pattern.	
	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade
[†] Belgium (Fl)	96 (0.9)	94 (2.2)	84 (2.1)	83 (2.4)	26 (2.5)	31 (2.9)
[†] Belgium (Fr)	93 (1.8)	96 (1.4)	87 (2.1)	84 (2.5)	13 (2.2)	22 (2.5)
Canada	91 (1.7)	97 (0.8)	78 (2.0)	82 (1.7)	21 (1.8)	33 (2.4)
Cyprus	73 (2.3)	83 (2.6)	66 (2.5)	69 (2.7)	11 (1.9)	20 (2.4)
Czech Republic	96 (0.9)	98 (0.6)	75 (2.8)	75 (2.4)	19 (2.3)	32 (3.4)
¹² England	94 (1.9)	95 (1.6)	84 (2.6)	86 (2.4)	20 (2.6)	42 (3.4)
France	93 (1.6)	92 (1.4)	80 (2.1)	80 (2.1)	12 (1.8)	18 (2.5)
Hong Kong	91 (1.8)	90 (2.1)	83 (2.7)	82 (1.9)	43 (2.8)	48 (2.7)
Hungary	93 (1.6)	93 (1.3)	84 (1.9)	91 (1.4)	20 (2.9)	34 (2.8)
Iceland	83 (2.5)	83 (3.7)	74 (3.5)	77 (3.6)	6 (1.7)	16 (2.7)
Iran, Islamic Rep.	88 (2.2)	95 (1.3)	64 (3.0)	65 (2.8)	2 (0.8)	12 (2.7)
Ireland	92 (1.6)	94 (1.3)	72 (2.2)	73 (2.3)	19 (2.0)	25 (2.6)
Japan	97 (0.6)	96 (0.8)	89 (1.4)	94 (0.8)	43 (2.2)	52 (2.2)
Korea	96 (1.2)	97 (0.9)	80 (2.6)	84 (2.1)	32 (2.8)	38 (2.6)
¹ Latvia (LSS)	93 (1.6)	96 (1.2)	67 (2.8)	76 (2.7)	13 (2.2)	17 (2.4)
¹ Lithuania	87 (2.0)	91 (1.9)	56 (3.4)	66 (3.2)	6 (1.6)	13 (2.2)
New Zealand	90 (1.9)	94 (1.2)	72 (2.5)	81 (2.0)	23 (2.5)	31 (2.5)
Norway	88 (2.1)	92 (1.5)	73 (3.0)	77 (2.3)	14 (2.4)	22 (2.4)
Portugal	89 (1.9)	94 (1.3)	62 (2.6)	71 (2.6)	6 (1.5)	13 (1.8)
Russian Federation	92 (1.5)	95 (1.2)	70 (1.8)	76 (2.3)	11 (1.5)	22 (2.0)
[†] Scotland	89 (1.7)	94 (1.1)	85 (1.9)	89 (1.8)	18 (2.0)	35 (2.8)
Singapore	93 (1.3)	95 (0.8)	79 (2.4)	83 (1.5)	37 (2.9)	50 (2.8)
Slovak Republic	90 (1.7)	92 (1.5)	67 (2.5)	73 (2.4)	15 (1.9)	27 (2.4)
Spain	89 (1.7)	93 (1.3)	71 (2.4)	80 (2.0)	17 (2.2)	22 (2.0)
Sweden	90 (1.7)	89 (1.4)	75 (2.5)	75 (2.1)	8 (1.6)	17 (2.0)
[†] Switzerland	95 (1.1)	95 (1.4)	80 (2.1)	86 (1.7)	27 (2.6)	38 (2.5)
[†] United States	90 (1.8)	93 (0.8)	73 (2.2)	75 (2.2)	18 (2.4)	25 (1.6)
Countries Not Satisfying Guidelines for Sample Participation Rates (See Appendix A for Details):						
Australia	91 (1.3)	93 (1.3)	76 (2.5)	80 (1.3)	26 (2.5)	32 (1.8)
Austria	95 (1.4)	95 (1.4)	91 (1.9)	91 (2.1)	27 (2.2)	35 (3.4)
Bulgaria	83 (3.5)	88 (3.4)	69 (4.5)	76 (3.5)	18 (4.3)	18 (3.5)
Netherlands	87 (2.4)	91 (1.9)	82 (2.8)	84 (2.5)	29 (2.9)	38 (3.8)
Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details):						
Colombia	44 (3.6)	55 (4.2)	45 (3.9)	46 (4.2)	7 (4.8)	11 (4.1)
^{†1} Germany	86 (2.1)	92 (1.6)	79 (2.9)	81 (2.4)	16 (2.4)	18 (2.6)
Romania	83 (2.0)	85 (2.0)	53 (2.9)	63 (2.6)	15 (2.0)	20 (2.4)
Slovenia	87 (2.0)	89 (1.6)	76 (2.2)	82 (2.4)	20 (2.4)	31 (3.2)
Countries With Unapproved Sampling Procedures at Classroom Level (See Appendix A for Details):						
Denmark	91 (1.6)	93 (1.8)	68 (2.7)	77 (2.9)	13 (2.0)	24 (3.4)
Greece	77 (2.2)	86 (1.6)	69 (2.1)	79 (2.2)	4 (1.0)	13 (2.1)
[†] South Africa	44 (2.7)	53 (3.3)	19 (2.5)	20 (2.5)	3 (0.9)	3 (1.3)
Thailand	94 (0.9)	96 (0.8)	78 (1.9)	86 (1.3)	19 (1.6)	26 (2.7)
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):						
[†] Israel	–	91 (1.4)	–	78 (2.7)	–	25 (3.4)
Kuwait	–	78 (4.1)	–	34 (3.9)	–	20 (4.0)

*Seventh and eighth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (–) indicates data are not available. Israel and Kuwait did not test at the seventh grade.

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1994-95.

Table 3.3 (Continued)**Percent Correct for Algebra Example Items
Lower and Upper Grades (Seventh and Eighth Grades*)**

Country	Example 15 Solve linear equation for x.		Example 16 Equivalent algebraic expressions.		Example 17 Expression representing number of hats.	
	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade
[†] Belgium (Fl)	84 (2.3)	80 (2.8)	69 (2.8)	69 (4.2)	41 (3.0)	53 (3.8)
[†] Belgium (Fr)	69 (3.4)	76 (2.5)	56 (3.7)	64 (2.7)	35 (3.5)	46 (3.1)
Canada	55 (2.6)	73 (2.6)	40 (2.3)	61 (2.1)	33 (2.5)	45 (2.7)
Cyprus	65 (3.4)	71 (3.2)	43 (2.6)	59 (2.9)	34 (2.9)	47 (3.0)
Czech Republic	81 (2.6)	86 (2.2)	69 (3.2)	75 (2.7)	56 (3.1)	70 (3.7)
¹² England	51 (3.2)	61 (3.4)	46 (3.6)	42 (3.6)	25 (3.2)	37 (3.0)
France	62 (2.6)	82 (2.3)	53 (2.8)	65 (2.5)	39 (2.7)	55 (2.8)
Hong Kong	87 (2.4)	92 (1.9)	72 (3.3)	79 (3.3)	64 (3.4)	65 (3.2)
Hungary	79 (2.1)	89 (1.7)	61 (2.7)	72 (2.4)	40 (3.2)	57 (3.0)
Iceland	45 (3.7)	56 (3.4)	35 (3.0)	59 (4.0)	11 (2.2)	14 (3.2)
Iran, Islamic Rep.	36 (4.5)	47 (3.7)	31 (3.3)	34 (3.2)	29 (3.2)	38 (3.8)
Ireland	65 (2.6)	72 (3.0)	39 (2.9)	53 (2.8)	44 (2.1)	51 (2.6)
Japan	85 (1.7)	90 (1.3)	60 (2.0)	75 (1.9)	48 (2.3)	57 (2.2)
Korea	87 (1.9)	92 (1.6)	56 (3.1)	65 (2.6)	60 (3.2)	64 (2.7)
¹ Latvia (LSS)	70 (3.1)	75 (2.5)	49 (3.3)	58 (3.0)	45 (3.2)	42 (3.3)
¹ Lithuania	66 (3.3)	72 (3.4)	48 (3.4)	56 (3.8)	39 (3.2)	46 (3.5)
New Zealand	56 (2.9)	69 (2.4)	40 (2.8)	55 (2.6)	27 (2.8)	38 (2.6)
Norway	32 (2.8)	52 (2.5)	42 (4.2)	52 (2.7)	13 (2.8)	23 (2.3)
Portugal	47 (2.6)	60 (2.2)	26 (2.9)	42 (2.9)	30 (2.6)	42 (2.3)
Russian Federation	84 (2.0)	88 (1.7)	61 (2.9)	75 (2.9)	54 (2.5)	58 (3.8)
[†] Scotland	40 (2.7)	62 (2.8)	53 (3.0)	53 (3.0)	18 (2.1)	36 (3.1)
Singapore	91 (1.7)	96 (0.9)	77 (2.2)	82 (2.0)	78 (2.4)	86 (1.7)
Slovak Republic	83 (1.8)	84 (2.1)	63 (3.1)	77 (2.6)	54 (2.8)	66 (2.6)
Spain	58 (2.8)	76 (2.3)	43 (2.5)	59 (2.7)	46 (2.4)	61 (2.3)
Sweden	42 (2.7)	51 (2.7)	37 (2.5)	51 (2.6)	16 (2.3)	20 (2.0)
¹ Switzerland	54 (2.3)	77 (2.2)	38 (2.5)	54 (2.7)	28 (2.4)	41 (3.1)
[†] United States	63 (3.8)	73 (2.3)	40 (2.8)	46 (2.5)	39 (2.9)	49 (2.3)
Countries Not Satisfying Guidelines for Sample Participation Rates (See Appendix A for Details):						
Australia	65 (2.5)	73 (1.6)	51 (2.7)	65 (1.8)	31 (2.3)	45 (2.0)
Austria	70 (2.8)	80 (2.1)	51 (2.7)	73 (2.8)	38 (2.9)	51 (3.1)
Bulgaria	82 (3.1)	84 (2.6)	69 (3.5)	72 (3.1)	64 (5.1)	64 (3.9)
Netherlands	49 (4.0)	65 (4.3)	33 (4.1)	51 (4.5)	27 (2.9)	45 (4.0)
Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details):						
Colombia	30 (3.3)	43 (3.7)	19 (3.6)	34 (4.5)	23 (3.5)	33 (3.7)
^{†1} Germany	62 (3.6)	79 (2.0)	43 (3.4)	57 (3.3)	27 (2.5)	41 (3.0)
Romania	70 (2.6)	77 (2.7)	57 (2.6)	64 (2.7)	45 (3.0)	52 (3.0)
Slovenia	74 (2.5)	86 (1.8)	55 (2.8)	75 (2.7)	43 (2.8)	55 (3.0)
Countries With Unapproved Sampling Procedures at Classroom Level (See Appendix A for Details):						
Denmark	53 (3.9)	70 (3.3)	31 (2.7)	36 (3.1)	16 (2.3)	29 (2.8)
Greece	62 (2.2)	75 (2.2)	40 (2.7)	57 (2.5)	29 (2.1)	36 (2.7)
[†] South Africa	38 (2.1)	39 (2.5)	25 (2.0)	33 (2.7)	21 (2.1)	19 (2.4)
Thailand	71 (2.4)	79 (2.2)	40 (2.5)	49 (3.1)	40 (2.6)	46 (2.6)
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):						
¹ Israel	–	86 (2.9)	–	70 (3.7)	–	73 (3.3)
Kuwait	–	50 (3.9)	–	29 (2.8)	–	27 (3.3)

*Seventh and eighth grades in most countries; See Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

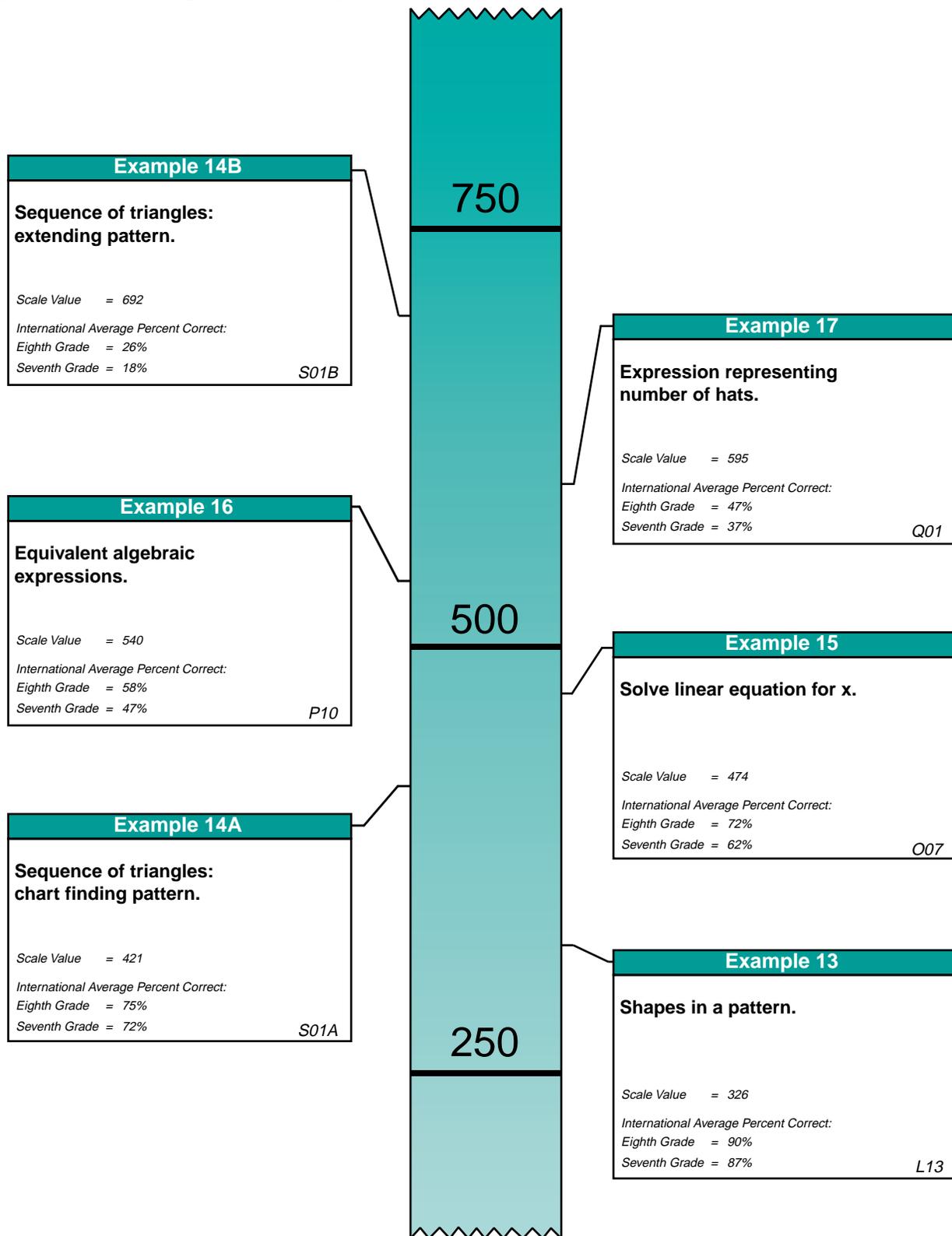
¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (–) indicates data are not available. Israel and Kuwait did not test at the seventh grade.

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1994-95.

Figure 3.3**International Difficulty Map for Algebra Example Items - Lower and Upper Grades (Seventh and Eighth Grades*)**

*Seventh and eighth grades in most countries; see Table 2 for information about the grades tested in each country.

NOTE: Each item was placed onto the TIMSS international mathematics scale based on students' performance in both grades. Items are shown at the point on the scale where students with that level of proficiency had a 65 percent probability of providing a correct response.

EXAMPLE ITEM 13
ALGEBRA

Shapes in a pattern

These shapes are arranged in a pattern.

○△○○△△○○○△△△

Which set of shapes is arranged in the same pattern?

- A. ★□★□★□★□★□★□
- B. □★□□★□□★□□□
- C. ★□★□★□★□★□★□
- D. □□★□★□□★□★

Performance Category: Knowing

EXAMPLE ITEM 14
ALGEBRA

Sequence of triangles

Here is a sequence of three similar triangles. All of the small triangles are congruent.

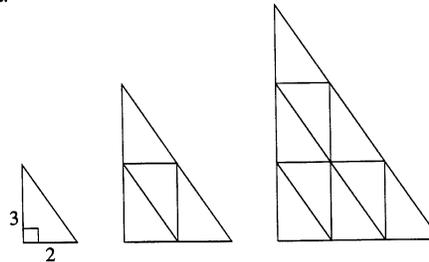


Figure 1 Figure 2 Figure 3

- a. Complete the chart by finding how many small triangles make up each figure.

Figure	Number of small triangles
1	1
2	4
3	9

- b. The sequence of similar triangles is extended to the 8th Figure. How many small triangles would be needed for Figure 8?

Handwritten work for part b:

$$9 + 7 + 9 + 11 + 13 + 15$$

Handwritten calculations:

$$18 \quad 29$$

$$\begin{array}{r} 2 \\ 18 \\ 18 \\ \hline 36 \\ 64 \end{array}$$

64 small triangles

Performance Category: Solving Problems

EXAMPLE ITEM 15
ALGEBRA**Solve linear equation for x**

If $3(x + 5) = 30$, then $x =$

- A. 2
- B. 5
- C. 10
- D. 95

Performance Category: Performing Routine Procedures

EXAMPLE ITEM 16
ALGEBRA**Equivalent algebraic expressions**

If m represents a positive number, which of these is equivalent to $m + m + m + m$?

- A. $m + 4$
- B. $4m$
- C. m^4
- D. $4(m + 1)$

Performance Category: Knowing

EXAMPLE ITEM 17
ALGEBRA
Expression representing number of hats

Juan has 5 fewer hats than Maria, and Clarissa has 3 times as many hats as Juan. If Maria has n hats, which of these represents the number of hats that Clarissa has?

- A. $5 - 3n$
- B. $3n$
- C. $n - 5$
- D. $3n - 5$
- E. $3(n - 5)$

Performance Category: Using Complex Procedures

WHAT HAVE STUDENTS LEARNED ABOUT DATA REPRESENTATION, ANALYSIS, AND PROBABILITY?

As illustrated by Example Items 18 through 23, the types of items in this content area required students to represent and analyze data using charts, tables, and graphs and to demonstrate their understanding of basic concepts underlying uncertainty and probability. The results for the example items are presented in Table 3.4. As shown in Figure 3.4, the international difficulty map for data representation, analysis, and probability indicates that the higher performing students were more likely to demonstrate the ability to apply concepts and integrate their understandings.

Example Item 18 asked students to read a chart of daily temperatures. Performance on reading the chart of temperatures was high in nearly all countries (international averages of 85% and 87%). Performance also was relatively high on Example Item 19 which required students to complete a pictograph (international averages of 79% and 81%).

Example Item 21, requiring students to read a line graph, posed a greater challenge for students in many countries. On average, 51% of the students at the seventh grade across countries and 58% at the eighth grade answered this question correctly. There were large differences in performance among countries. At the eighth grade, performance at 75% correct or better was achieved in Flemish-speaking Belgium (82%), France (81%), Japan (75%), Switzerland (77%), the Netherlands (76%), and Denmark (75%). Performance below 45% occurred in Cyprus (40%), Iran (25%), Colombia (20%), Romania (36%), South Africa (17%), and Kuwait (24%).

Example Items 20 and 22 assessed the area of probability. In general, students appeared to understand that the probability of picking the one red marble was highest for the fewest number of marbles (Example Item 20). The international averages were 73% and 76% at the seventh and eighth grades, respectively. Eighty-five percent or more of the students at both grades answered this question correctly in Belgium (Flemish), Canada, Hong Kong, Korea, and the Netherlands. In contrast, asking students to integrate their understanding of both cubes and probability proved to be more difficult for them (Example Item 22). The international averages of correct responses were 41% at the seventh grade and 47% at the eighth grade. Although the eighth-grade students performed quite well in Singapore (88%) and two-thirds or more answered correctly in Flemish-speaking Belgium (68%), Hong Kong (72%), Japan (75%), and Korea (68%), performance fell below 40% correct in a number of countries.

Example Item 23 required students to apply their mathematics understanding to an everyday situation — that of extracting and using appropriate information from a newspaper advertisement to determine which office space had the lower rent. Students were asked to show their work. Although the scoring approach provided information about partial solutions to the problem, the results reported herein for each country are for those students receiving complete credit for the item. That is, students indicated that Building A had the lower price and showed accurate computations to support this conclusion. Performance was quite low in most of the countries. Only in Singapore (55%) did more than half the eighth-grade students provide a complete solution to this problem, although performance in Japan (47%) and Korea (50%) also was higher than in other countries.

Table 3.4

**Percent Correct for Data Representation, Analysis, and Probability
Example Items
Lower and Upper Grades (Seventh and Eighth Grades*)**

Country	Example 18 Highest temperature on chart.		Example 19 Pictograph of number of students.		Example 20 Chance of picking red marble.	
	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade
[†] Belgium (Fl)	94 (1.4)	91 (2.5)	93 (1.2)	86 (3.8)	90 (1.9)	86 (1.9)
[†] Belgium (Fr)	92 (1.7)	90 (2.3)	84 (2.3)	82 (2.8)	83 (2.4)	85 (2.3)
Canada	90 (1.6)	92 (1.7)	91 (1.3)	89 (1.5)	85 (1.9)	90 (1.1)
Cyprus	72 (2.7)	78 (2.5)	75 (2.5)	82 (1.8)	63 (2.4)	68 (2.9)
Czech Republic	97 (1.0)	96 (0.8)	76 (2.4)	84 (2.3)	66 (2.6)	76 (2.8)
^{†2} England	89 (2.1)	91 (2.2)	87 (2.7)	92 (1.7)	81 (2.7)	86 (2.3)
France	89 (1.7)	90 (1.7)	85 (1.9)	88 (1.6)	82 (2.4)	82 (2.3)
Hong Kong	85 (1.9)	79 (2.8)	86 (2.0)	81 (2.0)	85 (2.5)	89 (1.6)
Hungary	92 (1.5)	91 (1.4)	83 (2.0)	87 (1.7)	77 (2.3)	82 (2.1)
Iceland	88 (2.0)	90 (2.2)	87 (2.8)	87 (2.9)	76 (3.0)	77 (2.8)
Iran, Islamic Rep.	72 (3.1)	75 (2.9)	52 (3.3)	67 (2.9)	31 (5.4)	37 (3.1)
Ireland	90 (1.5)	92 (1.6)	84 (2.0)	89 (1.8)	76 (2.3)	82 (2.1)
Japan	94 (1.0)	93 (1.1)	93 (0.9)	94 (1.0)	81 (1.7)	83 (1.4)
Korea	82 (2.4)	85 (1.8)	92 (1.7)	90 (1.6)	86 (2.0)	91 (1.6)
¹ Latvia (LSS)	80 (2.6)	86 (2.2)	72 (2.4)	82 (1.9)	51 (2.8)	60 (3.0)
¹ Lithuania	74 (3.2)	87 (2.1)	59 (3.3)	75 (2.8)	56 (3.1)	68 (2.9)
New Zealand	91 (1.9)	93 (1.3)	87 (1.9)	92 (1.4)	74 (2.3)	82 (1.7)
Norway	88 (2.0)	92 (1.5)	85 (2.3)	86 (1.9)	79 (2.8)	85 (1.7)
Portugal	84 (2.0)	90 (1.6)	78 (2.1)	86 (1.8)	60 (2.4)	67 (2.3)
Russian Federation	84 (2.2)	91 (1.5)	77 (2.2)	78 (2.2)	63 (2.8)	70 (2.5)
[†] Scotland	89 (1.7)	91 (1.7)	83 (1.8)	88 (1.7)	77 (2.4)	82 (2.0)
Singapore	80 (2.1)	88 (1.4)	92 (1.3)	94 (1.1)	82 (2.0)	81 (1.9)
Slovak Republic	90 (1.5)	93 (1.4)	79 (2.0)	80 (2.0)	70 (2.4)	70 (2.6)
Spain	86 (1.7)	88 (1.7)	77 (2.5)	86 (1.7)	80 (2.2)	83 (2.0)
Sweden	93 (1.5)	94 (1.3)	86 (1.9)	87 (1.5)	84 (1.7)	81 (1.9)
[†] Switzerland	94 (1.1)	92 (1.8)	86 (2.3)	88 (2.1)	81 (2.5)	86 (1.4)
[†] United States	89 (1.7)	90 (1.1)	87 (1.5)	89 (1.2)	82 (1.9)	86 (1.2)
Countries Not Satisfying Guidelines for Sample Participation Rates (See Appendix A for Details):						
Australia	94 (1.1)	92 (1.4)	91 (1.4)	88 (1.4)	79 (2.1)	84 (1.6)
Austria	90 (1.5)	91 (1.9)	84 (2.5)	87 (2.1)	77 (2.6)	82 (2.3)
Bulgaria	82 (3.5)	81 (2.8)	74 (3.6)	75 (4.1)	77 (3.6)	85 (3.8)
Netherlands	92 (2.0)	89 (2.4)	89 (2.3)	87 (3.6)	89 (2.1)	91 (1.9)
Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details):						
Colombia	66 (2.9)	71 (4.0)	53 (3.6)	64 (4.2)	40 (3.4)	47 (4.0)
^{†1} Germany	89 (2.1)	87 (2.2)	83 (2.0)	82 (2.7)	78 (2.1)	83 (2.2)
Romania	72 (3.1)	69 (2.8)	64 (3.0)	64 (2.7)	52 (2.8)	52 (2.7)
Slovenia	93 (1.3)	95 (1.2)	82 (1.8)	77 (2.0)	81 (2.1)	85 (2.2)
Countries With Unapproved Sampling Procedures at Classroom Level (See Appendix A for Details):						
Denmark	93 (1.8)	92 (2.1)	84 (2.7)	88 (2.2)	76 (2.5)	83 (2.2)
Greece	78 (2.2)	85 (1.7)	63 (2.7)	77 (2.5)	61 (2.2)	71 (1.9)
[†] South Africa	48 (2.7)	55 (2.6)	17 (2.5)	17 (3.1)	30 (2.5)	28 (2.8)
Thailand	83 (1.8)	86 (1.5)	93 (1.3)	94 (1.0)	74 (2.0)	76 (1.9)
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):						
[†] Israel	–	89 (2.2)	–	87 (3.3)	–	77 (3.2)
Kuwait	–	82 (2.7)	–	29 (4.6)	–	53 (4.4)

*Seventh and eighth grades in most countries; see Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (–) indicates data are not available. Israel and Kuwait did not test at the seventh grade.

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1994-95.

Table 3.4 (Continued)

**Percent Correct for Data Representation, Analysis, and Probability
Example Items
Lower and Upper Grades (Seventh and Eighth Grades*)**

Country	Example 21 Speed of car from graph.		Example 22 Number of red cube faces.		Example 23 Price of renting office space.	
	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade
[†] Belgium (Fl)	76 (2.6)	82 (3.8)	73 (3.1)	68 (2.7)	25 (2.3)	23 (1.9)
[†] Belgium (Fr)	60 (2.8)	64 (3.8)	55 (3.2)	61 (3.8)	14 (1.5)	20 (2.5)
Canada	55 (2.2)	66 (1.9)	49 (2.6)	57 (2.2)	16 (1.5)	24 (1.7)
Cyprus	41 (2.6)	40 (3.2)	37 (2.8)	46 (3.0)	5 (0.7)	8 (1.6)
Czech Republic	57 (3.1)	71 (2.8)	39 (3.2)	36 (3.2)	18 (1.8)	28 (2.6)
^{†2} England	66 (2.8)	69 (3.1)	36 (3.2)	39 (3.1)	12 (1.5)	20 (2.0)
France	75 (2.1)	81 (2.5)	43 (3.0)	54 (3.0)	16 (1.5)	26 (2.1)
Hong Kong	65 (2.9)	65 (2.5)	70 (3.2)	72 (2.7)	25 (2.3)	37 (2.5)
Hungary	57 (3.0)	61 (2.7)	43 (2.7)	55 (2.8)	11 (1.2)	20 (1.6)
Iceland	37 (3.6)	56 (4.3)	36 (2.9)	57 (4.2)	6 (1.3)	15 (1.8)
Iran, Islamic Rep.	17 (3.2)	25 (2.8)	26 (2.4)	24 (3.9)	1 (0.4)	1 (0.4)
Ireland	50 (2.6)	63 (2.4)	58 (2.4)	64 (3.3)	18 (1.6)	25 (2.3)
Japan	71 (1.9)	75 (1.8)	69 (2.1)	75 (1.6)	38 (1.5)	47 (1.5)
Korea	61 (2.5)	67 (2.6)	66 (2.7)	68 (3.2)	38 (2.1)	50 (1.8)
¹ Latvia (LSS)	43 (3.2)	57 (3.0)	22 (2.1)	28 (3.0)	5 (1.2)	9 (1.2)
¹ Lithuania	47 (3.0)	53 (3.3)	18 (2.7)	22 (2.9)	3 (0.9)	7 (1.2)
New Zealand	51 (2.6)	66 (2.6)	37 (2.6)	52 (2.4)	15 (1.5)	22 (2.0)
Norway	58 (3.4)	73 (2.3)	42 (3.5)	57 (2.6)	16 (1.8)	23 (1.6)
Portugal	38 (2.4)	49 (2.6)	18 (1.9)	21 (1.9)	4 (0.7)	8 (0.9)
Russian Federation	49 (3.2)	49 (3.0)	29 (2.7)	33 (2.6)	11 (1.3)	14 (1.7)
[†] Scotland	60 (3.2)	70 (2.7)	36 (2.9)	48 (3.3)	12 (1.4)	20 (2.3)
Singapore	57 (2.5)	67 (2.0)	80 (2.1)	88 (1.7)	49 (2.6)	55 (2.0)
Slovak Republic	42 (2.5)	56 (2.8)	37 (2.4)	43 (2.9)	10 (1.3)	15 (1.7)
Spain	39 (2.7)	47 (2.6)	24 (2.1)	34 (2.6)	6 (0.8)	15 (1.3)
Sweden	62 (3.0)	74 (2.3)	45 (3.1)	55 (2.7)	18 (1.9)	23 (1.7)
¹ Switzerland	67 (2.9)	77 (2.3)	55 (2.7)	64 (3.0)	16 (1.5)	26 (1.5)
[†] United States	59 (2.9)	72 (1.9)	37 (3.3)	47 (3.0)	15 (2.2)	18 (1.6)
Countries Not Satisfying Guidelines for Sample Participation Rates (See Appendix A for Details):						
Australia	62 (2.3)	72 (1.7)	49 (2.8)	53 (2.2)	18 (1.6)	22 (1.3)
Austria	59 (2.9)	74 (2.2)	47 (2.7)	54 (3.3)	17 (1.6)	25 (1.8)
Bulgaria	35 (3.7)	49 (4.3)	38 (4.0)	46 (5.7)	9 (1.5)	6 (1.4)
Netherlands	70 (3.4)	76 (3.8)	60 (3.3)	62 (3.6)	14 (2.2)	24 (2.6)
Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details):						
Colombia	16 (2.2)	20 (2.7)	16 (2.6)	15 (2.0)	1 (0.4)	1 (0.5)
^{†1} Germany	68 (2.8)	69 (3.2)	50 (3.8)	45 (3.5)	14 (1.9)	14 (1.7)
Romania	31 (2.6)	36 (2.8)	20 (2.2)	33 (2.8)	7 (1.2)	12 (1.7)
Slovenia	57 (2.8)	57 (2.9)	33 (2.7)	42 (2.7)	12 (1.5)	20 (1.6)
Countries With Unapproved Sampling Procedures at Classroom Level (See Appendix A for Details):						
Denmark	60 (4.0)	75 (2.8)	36 (3.9)	46 (2.9)	12 (2.0)	22 (2.2)
Greece	29 (2.1)	48 (2.8)	34 (2.1)	38 (2.6)	9 (1.2)	13 (1.2)
[†] South Africa	17 (1.9)	17 (2.3)	12 (1.7)	15 (1.9)	2 (0.8)	2 (1.1)
Thailand	48 (2.4)	56 (2.7)	40 (2.8)	55 (2.9)	13 (1.7)	21 (2.5)
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):						
¹ Israel	–	56 (4.1)	–	53 (4.4)	–	15 (2.5)
Kuwait	–	24 (3.9)	–	19 (3.7)	–	4 (1.2)

*Seventh and eighth grades in most countries; see Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

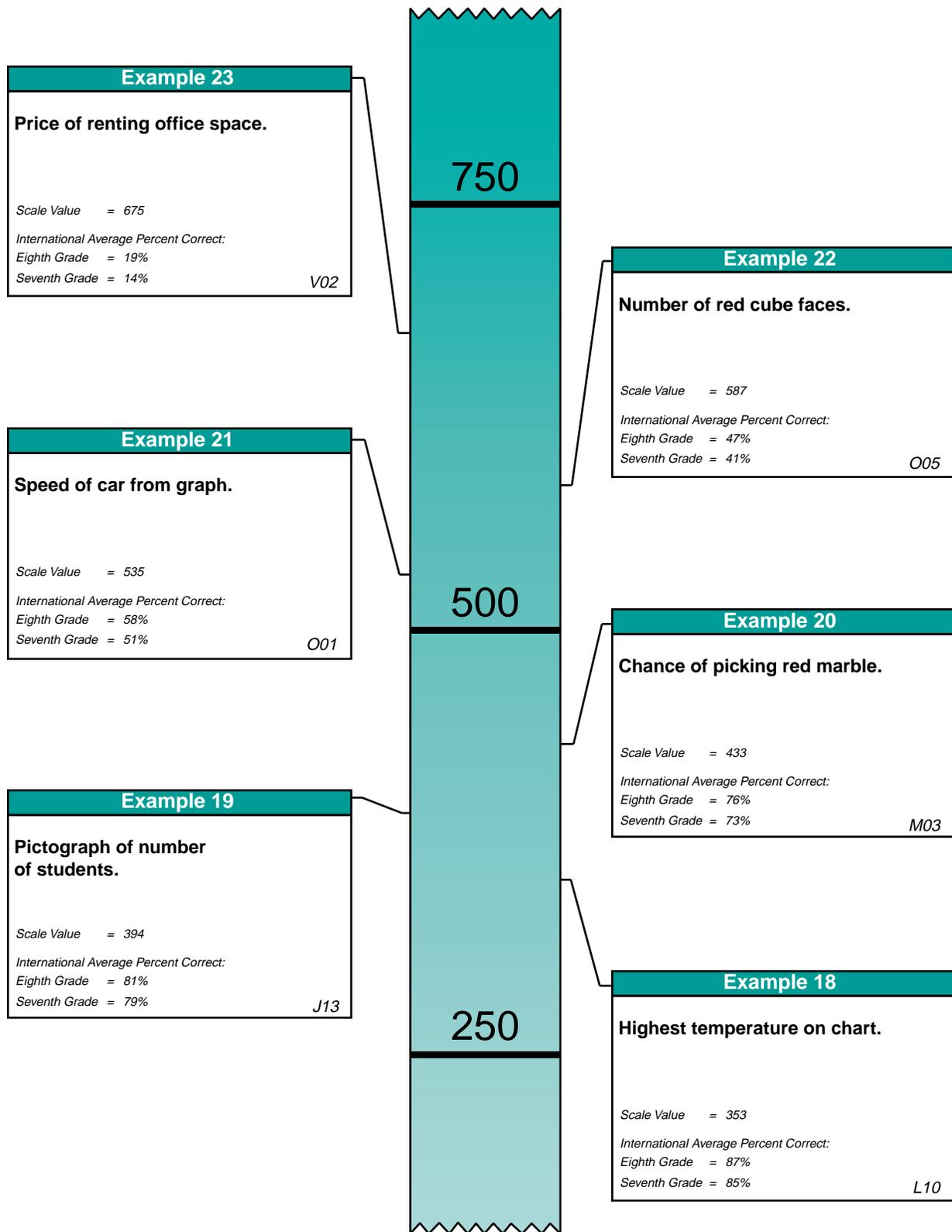
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (–) indicates data are not available. Israel and Kuwait did not test at the seventh grade.

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1994-95.

Figure 3.4

**International Difficulty Map for Data Representation, Analysis, and Probability
Example Items
Lower and Upper Grades (Seventh and Eighth Grades*)**



*Seventh and eighth grades in most countries; see Table 2 for information about the grades tested in each country.
NOTE: Each item was placed onto the TIMSS international mathematics scale based on students' performance in both grades. Items are shown at the point on the scale where students with that level of proficiency had a 65 percent probability of providing a correct response.

EXAMPLE ITEM 18

DATA REPRESENTATION, ANALYSIS & PROBABILITY

Highest temperature on chart

This chart shows temperature readings made at different times on four days.

TEMPERATURES					
	6 a.m.	9 a.m.	Noon	3 p.m.	8 p.m.
Monday	15°	17°	20°	21°	19°
Tuesday	15°	15°	15°	10°	9°
Wednesday	8°	10°	14°	13°	15°
Thursday	8°	11°	14°	17°	20°

When was the highest temperature recorded?

- A. Noon on Monday
- B. 3 p.m. on Monday
- C. Noon on Tuesday
- D. 3 p.m. on Wednesday

Performance Category: Using Complex Procedures

EXAMPLE ITEM 19

DATA REPRESENTATION, ANALYSIS & PROBABILITY

Pictograph of number of students

The table shows the number of students in the 7th and 8th grades in a given school.

Grade	Number of Students
7	60
8	55

Complete the Grade 8 row in the pictograph below to represent the number of students in each grade.

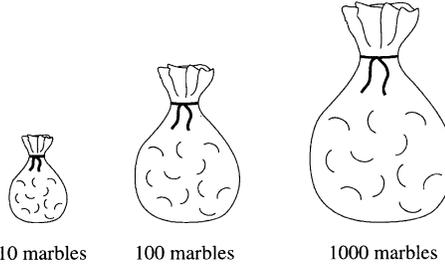
One ☺ represents 10 students

Grade 7	☺☺☺☺☺☺
Grade 8	☺☺☺☺☺☺

Performance Category: Using Complex Procedures

EXAMPLE ITEM 20
DATA REPRESENTATION, ANALYSIS & PROBABILITY**Chance of picking red marble**

There is only one red marble in each of these bags.



10 marbles

100 marbles

1000 marbles

Without looking in the bags, you are to pick a marble out of one of the bags. Which bag would give you the greatest chance of picking the red marble?

- A. The bag with 10 marbles
- B. The bag with 100 marbles
- C. The bag with 1000 marbles
- D. All bags would give the same chance.

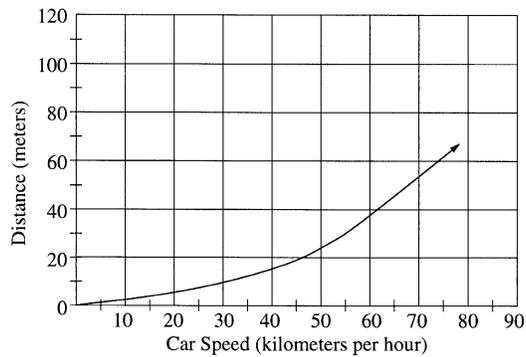
Performance Category: Solving Problems

EXAMPLE ITEM 21

DATA REPRESENTATION, ANALYSIS & PROBABILITY

Speed of car from graph

The graph shows the distance traveled before coming to a stop after the brakes are applied for a typical car traveling at different speeds.



A car traveling on a highway stopped 30 m after the brakes were applied. About how fast was the car traveling?

- A. 48 km per hour
- B. 55 km per hour
- C. 70 km per hour
- D. 160 km per hour

Performance Category: Solving Problems

EXAMPLE ITEM 22

DATA REPRESENTATION, ANALYSIS & PROBABILITY

Number of red cube faces

Each of the six faces of a certain cube is painted either red or blue. When the cube is tossed, the probability of the cube landing with a red face up is $\frac{2}{3}$.

How many faces are red?

- A. One
- B. Two
- C. Three
- D. Four
- E. Five

Performance Category: Solving Problems

EXAMPLE ITEM 23

DATA REPRESENTATION, ANALYSIS & PROBABILITY

Price of renting office space

The following two advertisements appeared in a newspaper in a country where the units of currency are *zeds*.

BUILDING A

Office space available

85 - 95 square meters

475 *zeds* per month

100 - 120 square meters

800 *zeds* per month

BUILDING B

Office space available

35 - 260 square meters

90 *zeds* per square meter
per year

If a company is interested in renting an office of 110 square meters in that country for a year, at which office building, A or B, should they rent the office in order to get the lower price? Show your work.

$$\begin{aligned} \text{Price of Renting } \alpha \text{ in Building A} &= 800 \times 12 \\ \text{in a year} &= 9600 \text{ (zeds)} \end{aligned}$$

$$\begin{aligned} \text{Price of Renting in Building B} &= 110 \times 90 \\ \text{in a year} &= 9900 \text{ (zeds)} \end{aligned}$$

$$\therefore 9600 < 9900$$

\therefore They should rent the office at Building A in order to get the lower price.

Performance Category: Solving Problems

WHAT HAVE STUDENTS LEARNED ABOUT MEASUREMENT?

The measurement items focused on students' understanding of units of length, weight, time, area, and volume as well as on interpreting scales of measures. Table 3.5 contains the percent-correct results for the example items in measurement, numbered Example Items 24 through 29. The international difficulty map for the measurement items (Figure 3.5) indicates that only the students with higher-than-average mathematics scores internationally were likely to demonstrate an ability to use measurement skills in situations involving several steps.

A more detailed look at performance on the example items suggests that students in many countries had a solid grasp of a variety of measuring units and how to interpret them. Students in most countries were able to read the weight shown on the scale (Example Item 24). The international averages on this item were 83% at the seventh grade and 87% at the eighth grade. Students also did relatively well on Example

Item 25 about pacing off the width of a room (on average, 69% and 74% at the seventh and eighth grades). This item required some thought to understand that the longer the paces, the fewer required to cross the room. The most prevalent misconception was to indicate that the greatest number of paces was related to the longest pace.

Example Item 26 required familiarity with the number of degrees in circles or parts of circles to identify the angle closest to 30 degrees. On average, it was answered correctly by 62% and 64% of the seventh- and eighth-grade students, respectively. For this item, the pattern of increased performance between the grades was fairly inconsistent, with a number of countries having the same or lower performance at the eighth as at the seventh grade.

Internationally, approximately half the students at the seventh and eighth grades (on average, 49% and 52%) were able to determine 10.5 cm as the length of the pencil (Example Item 27). Performance was generally consistent across most countries, although at the eighth grade, students did particularly well in Switzerland (73%), Austria (73%), and Germany (72%). They had the most difficulty in South Africa (17%).

Example Item 28 was a two-part task that first required students to actually draw a new rectangle whose length was one and one-half times the length of a given rectangle and whose width was half the width of that rectangle. All correctly drawn and labeled 9 cm by 2 cm rectangles were given full credit. In the second part of the item, students were asked to determine the ratio of the area of the new rectangle to the area of the one shown. In most countries, students had considerable difficulty with the first part of this multifaceted task, and even more trouble with the second part (even though the scoring for full credit permitted correct ratios based on incorrect drawings). On average, just 24% of the seventh-grade students and 31% of those at eighth grade provided a correct drawing of the new rectangle. In only two countries did at least half the eighth-grade students correctly draw the new rectangle, Korea (54%) and Austria (51%). Fewer than 20% were successful in Iceland (18%), the United States (16%), Colombia (5%), South Africa (4%), and Kuwait (10%). Internationally, the second part of the item was very difficult. On average, just 6% and 10% of the students at the two grades provided a correct ratio between the newly drawn and given rectangles.

Table 3.5

Percent Correct for Measurement Example Items Lower and Upper Grades (Seventh and Eighth Grades*)

Country	Example 24 Weight shown on scale.		Example 25 Measuring the width of a room.		Example 26 Angle closest to 30 degrees.	
	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade
[†] Belgium (Fl)	95 (1.3)	98 (0.7)	86 (2.1)	86 (2.7)	64 (2.6)	64 (3.2)
[†] Belgium (Fr)	92 (1.8)	89 (2.7)	81 (2.7)	84 (2.0)	73 (3.0)	67 (2.7)
Canada	88 (1.9)	90 (1.6)	60 (2.7)	70 (2.3)	62 (2.7)	65 (2.1)
Cyprus	67 (2.4)	72 (2.4)	54 (3.1)	63 (2.9)	60 (2.7)	64 (2.8)
Czech Republic	89 (1.8)	92 (1.7)	81 (2.1)	94 (1.4)	76 (2.9)	76 (3.0)
¹² England	85 (2.3)	94 (1.7)	62 (3.0)	73 (3.5)	63 (3.1)	62 (2.9)
France	93 (1.8)	94 (1.5)	79 (2.0)	81 (2.6)	64 (2.6)	76 (2.5)
Hong Kong	92 (1.5)	91 (1.7)	70 (2.9)	72 (2.8)	69 (2.6)	68 (2.3)
Hungary	92 (1.4)	92 (1.5)	62 (2.6)	59 (2.6)	71 (2.3)	77 (2.3)
Iceland	86 (2.2)	88 (2.2)	71 (3.6)	80 (4.0)	76 (2.6)	61 (4.4)
Iran, Islamic Rep.	61 (2.7)	71 (2.9)	40 (3.3)	57 (3.3)	52 (3.1)	63 (2.7)
Ireland	83 (2.2)	91 (1.7)	81 (2.1)	83 (2.0)	54 (2.6)	63 (2.6)
Japan	94 (1.0)	97 (0.6)	81 (1.7)	86 (1.3)	77 (2.0)	76 (1.8)
Korea	94 (1.3)	95 (1.2)	73 (2.8)	77 (2.2)	77 (2.5)	76 (2.2)
¹ Latvia (LSS)	82 (2.5)	84 (2.2)	78 (2.6)	91 (1.5)	64 (2.9)	65 (3.0)
¹ Lithuania	77 (2.4)	84 (2.2)	64 (3.3)	74 (3.4)	60 (3.1)	63 (2.9)
New Zealand	86 (1.9)	91 (1.4)	57 (3.3)	69 (2.3)	55 (2.8)	63 (2.4)
Norway	85 (2.1)	88 (1.7)	73 (2.9)	79 (2.2)	70 (3.0)	70 (2.0)
Portugal	81 (2.1)	84 (2.0)	73 (2.5)	79 (2.2)	48 (2.4)	48 (2.8)
Russian Federation	83 (2.2)	92 (1.3)	81 (2.2)	89 (1.5)	71 (2.4)	72 (2.8)
[†] Scotland	86 (1.8)	92 (1.5)	58 (3.0)	66 (3.0)	53 (2.7)	58 (2.7)
Singapore	93 (1.1)	96 (0.9)	70 (3.0)	77 (2.3)	73 (2.4)	73 (1.9)
Slovak Republic	88 (1.7)	88 (1.6)	82 (1.8)	88 (1.7)	79 (1.9)	74 (2.4)
Spain	73 (2.4)	83 (1.8)	74 (2.1)	81 (1.7)	56 (2.9)	59 (2.3)
Sweden	87 (1.6)	92 (1.3)	82 (2.0)	86 (1.8)	57 (2.6)	61 (2.5)
¹ Switzerland	92 (1.6)	97 (1.1)	90 (1.5)	87 (1.6)	51 (2.7)	73 (2.4)
[†] United States	83 (1.9)	87 (1.7)	36 (3.4)	48 (2.6)	55 (1.9)	57 (1.7)
Countries Not Satisfying Guidelines for Sample Participation Rates (See Appendix A for Details):						
Australia	89 (1.7)	94 (0.9)	63 (2.8)	70 (1.9)	63 (1.6)	64 (2.3)
Austria	88 (1.6)	90 (2.2)	80 (2.9)	86 (2.3)	80 (2.6)	74 (3.1)
Bulgaria	80 (2.9)	87 (4.4)	82 (3.2)	77 (3.4)	62 (4.0)	78 (3.3)
Netherlands	94 (1.9)	97 (1.1)	85 (2.4)	82 (3.0)	52 (4.7)	64 (3.3)
Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details):						
Colombia	53 (4.3)	58 (4.5)	45 (3.6)	55 (3.8)	32 (3.6)	37 (3.6)
^{††} Germany	93 (1.6)	94 (1.6)	79 (2.3)	79 (2.4)	65 (2.6)	63 (2.8)
Romania	72 (2.5)	74 (2.3)	65 (2.8)	70 (2.9)	58 (2.8)	59 (2.9)
Slovenia	89 (1.6)	95 (1.3)	87 (2.0)	90 (1.7)	80 (2.4)	77 (2.6)
Countries With Unapproved Sampling Procedures at Classroom Level (See Appendix A for Details):						
Denmark	88 (2.3)	88 (1.6)	75 (2.7)	80 (2.6)	61 (2.8)	69 (3.1)
Greece	79 (1.8)	86 (1.7)	61 (2.1)	70 (2.2)	56 (2.5)	64 (2.3)
[†] South Africa	49 (2.8)	52 (2.5)	18 (2.1)	23 (2.7)	33 (2.5)	34 (2.5)
Thailand	90 (1.4)	92 (1.1)	72 (2.5)	81 (1.8)	70 (2.2)	78 (1.7)
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):						
[†] Israel	–	86 (3.5)	–	79 (3.3)	–	50 (4.2)
Kuwait	–	58 (2.5)	–	39 (3.6)	–	49 (3.7)

*Seventh and eighth grades in most countries; see Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

()Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (–) indicates data are not available. Israel and Kuwait did not test at the seventh grade.

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1994-95.

Table 3.5 (Continued)**Percent Correct for Measurement Example Items
Lower and Upper Grades (Seventh and Eighth Grades*)**

Country	Example 27 Approximate length of pencil.		Example 28A New rectangle: Draw from ratio of sides.		Example 28B New rectangle: Ratio of areas.	
	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade
[†] Belgium (Fl)	72 (2.5)	69 (3.3)	47 (2.4)	48 (2.2)	7 (1.1)	9 (1.2)
[†] Belgium (Fr)	45 (3.7)	57 (3.7)	40 (2.6)	43 (2.5)	6 (1.4)	5 (1.1)
Canada	50 (2.9)	53 (2.0)	21 (1.5)	27 (1.7)	8 (0.7)	17 (1.2)
Cyprus	35 (2.9)	40 (3.4)	27 (2.0)	35 (2.1)	11 (1.5)	20 (1.8)
Czech Republic	63 (2.6)	67 (2.6)	27 (1.8)	36 (2.4)	5 (1.0)	13 (2.0)
¹² England	44 (3.7)	52 (3.0)	21 (1.9)	28 (2.1)	8 (1.1)	12 (1.9)
France	55 (2.9)	61 (2.6)	34 (2.3)	43 (2.2)	2 (0.5)	6 (0.9)
Hong Kong	59 (2.8)	60 (3.2)	39 (2.8)	46 (2.8)	17 (1.7)	25 (2.4)
Hungary	56 (2.9)	58 (2.6)	37 (1.9)	43 (2.1)	3 (0.6)	9 (0.9)
Iceland	27 (3.6)	27 (2.6)	11 (1.4)	18 (2.3)	1 (0.6)	5 (1.4)
Iran, Islamic Rep.	34 (2.9)	34 (3.3)	13 (2.0)	24 (2.0)	4 (1.1)	8 (1.4)
Ireland	40 (3.1)	52 (2.4)	26 (2.1)	35 (2.5)	18 (1.7)	20 (1.8)
Japan	52 (2.2)	64 (2.3)	–	–	–	–
Korea	56 (2.6)	60 (2.7)	48 (2.2)	54 (2.1)	31 (2.1)	39 (2.5)
¹ Latvia (LSS)	56 (2.5)	60 (2.5)	29 (2.3)	31 (2.3)	5 (1.2)	6 (1.4)
¹ Lithuania	37 (3.5)	41 (3.1)	14 (1.8)	24 (2.1)	0 (0.2)	6 (1.0)
New Zealand	48 (2.9)	52 (2.7)	17 (1.8)	27 (1.7)	3 (0.5)	8 (1.4)
Norway	52 (4.8)	62 (2.4)	21 (2.2)	32 (1.7)	2 (0.4)	2 (0.5)
Portugal	37 (3.3)	43 (2.7)	14 (1.3)	22 (1.8)	2 (0.6)	2 (0.5)
Russian Federation	51 (2.4)	59 (3.1)	27 (1.8)	39 (2.8)	7 (1.4)	17 (2.0)
[†] Scotland	39 (2.4)	45 (3.0)	19 (1.7)	27 (2.7)	3 (0.7)	12 (2.2)
Singapore	62 (2.6)	64 (2.3)	–	–	–	–
Slovak Republic	55 (2.7)	63 (2.8)	29 (1.8)	35 (2.1)	10 (1.3)	15 (1.5)
Spain	43 (3.0)	52 (2.6)	18 (1.6)	28 (1.7)	1 (0.4)	2 (0.4)
Sweden	61 (2.9)	67 (2.0)	18 (1.5)	30 (1.9)	6 (0.9)	11 (1.2)
¹ Switzerland	70 (2.5)	73 (2.6)	37 (2.4)	47 (1.9)	3 (0.5)	7 (1.0)
[†] United States	46 (2.7)	45 (2.2)	11 (1.4)	16 (1.6)	10 (1.6)	10 (0.9)
Countries Not Satisfying Guidelines for Sample Participation Rates (See Appendix A for Details):						
Australia	49 (2.2)	55 (1.9)	22 (1.5)	31 (1.6)	8 (0.9)	15 (1.2)
Austria	66 (3.0)	73 (2.5)	41 (2.0)	51 (2.8)	4 (1.0)	8 (1.3)
Bulgaria	43 (4.6)	45 (4.5)	35 (4.1)	27 (3.7)	9 (2.1)	10 (3.1)
Netherlands	68 (3.2)	62 (3.3)	31 (2.5)	40 (3.2)	6 (1.2)	8 (1.5)
Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details):						
Colombia	30 (2.9)	29 (2.5)	3 (0.8)	5 (1.0)	0 (0.0)	0 (0.2)
^{†1} Germany	70 (2.2)	72 (3.0)	28 (2.2)	34 (2.6)	2 (0.5)	4 (0.8)
Romania	40 (2.6)	41 (2.6)	23 (2.0)	28 (2.1)	10 (1.6)	15 (1.9)
Slovenia	60 (2.6)	70 (2.8)	26 (2.0)	37 (2.3)	5 (1.3)	10 (1.4)
Countries With Unapproved Sampling Procedures at Classroom Level (See Appendix A for Details):						
Denmark	49 (3.6)	52 (3.2)	16 (1.8)	24 (2.1)	3 (0.8)	5 (1.0)
Greece	28 (2.4)	33 (2.5)	15 (1.4)	23 (1.8)	4 (0.7)	12 (1.3)
[†] South Africa	20 (1.9)	17 (2.1)	4 (0.9)	4 (1.3)	0 (0.2)	0 (0.2)
Thailand	49 (2.2)	57 (2.5)	16 (1.7)	20 (1.7)	9 (2.1)	12 (1.5)
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):						
¹ Israel	–	44 (4.4)	–	48 (3.1)	–	7 (1.7)
Kuwait	–	31 (5.4)	–	10 (2.7)	–	6 (2.5)

*Seventh and eighth grades in most countries; see Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

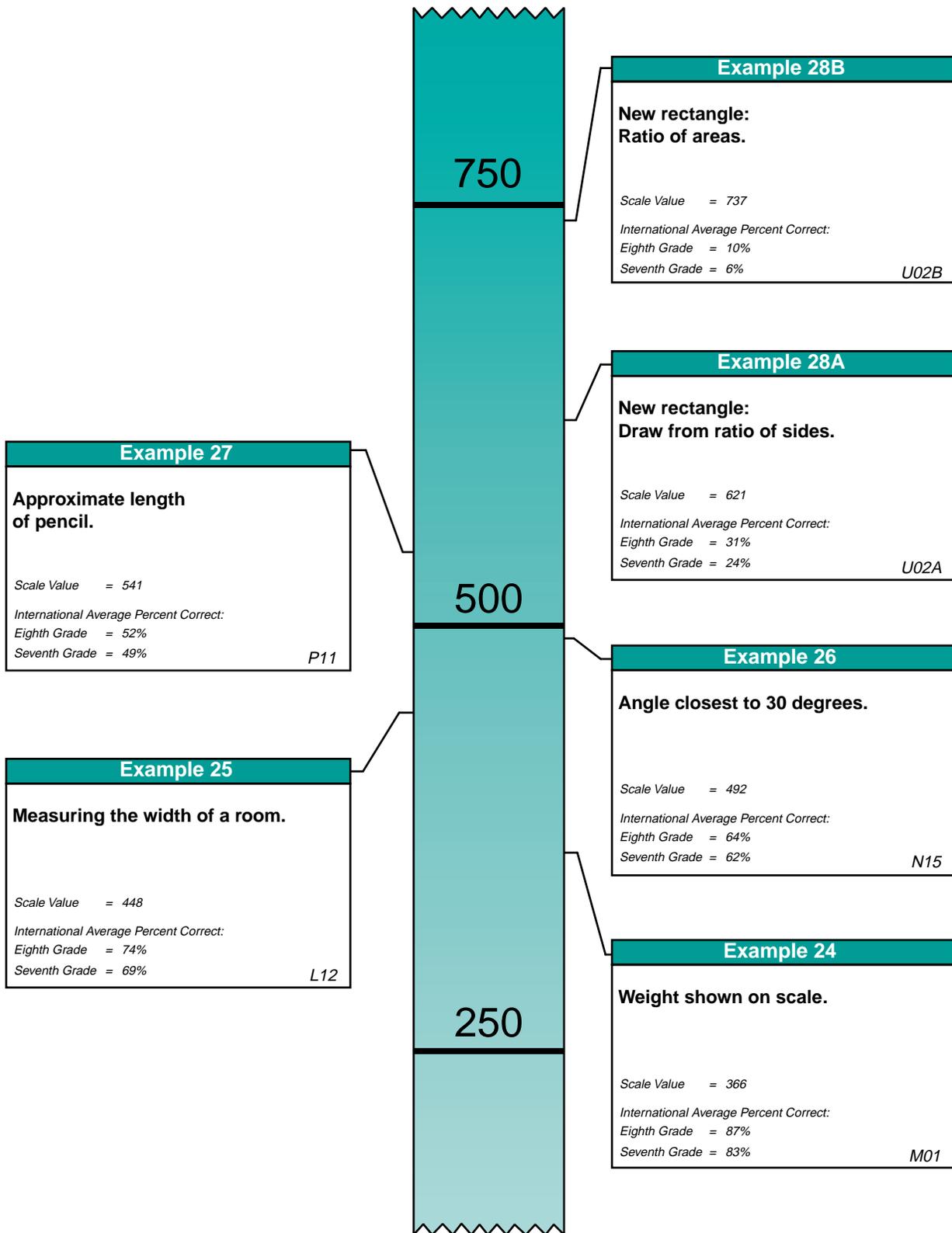
²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (–) indicates data are not available. Israel and Kuwait did not test at the seventh grade. Internationally comparable data are unavailable for Japan and Singapore on Examples 28A & 28B.

Figure 3.5

**International Difficulty Map for Measurement Example Items
Lower and Upper Grades (Seventh and Eighth Grades*)**



*Seventh and eighth grades in most countries; see Table 2 for information about the grades tested in each country.

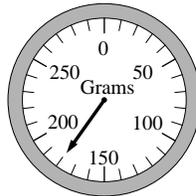
NOTE: Each item was placed onto the TIMSS international mathematics scale based on students' performance in both grades. Items are shown at the point on the scale where students with that level of proficiency had a 65 percent probability of providing a correct response.

EXAMPLE ITEM 24
MEASUREMENT

Weight shown on scale

What is the weight (mass) shown on the scale?

- A. 153 g
- B. 160 g
- C. 165 g
- D. 180 g



Performance Category: Knowing

EXAMPLE ITEM 25
MEASUREMENT

Measuring the width of a room

Four children measured the width of a room by counting how many paces it took them to cross it. The chart shows their measurements.

Who had the longest pace?

- A. Stephen
- B. Erlane
- C. Ana
- D. Carlos

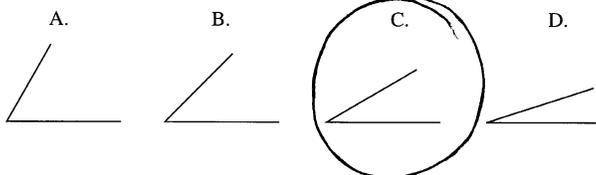
Name	Number of Paces
Stephen	10
Erlane	8
Ana	9
Carlos	7

Performance Category: Solving Problems

EXAMPLE ITEM 26
MEASUREMENT

Angle closest to 30 degrees

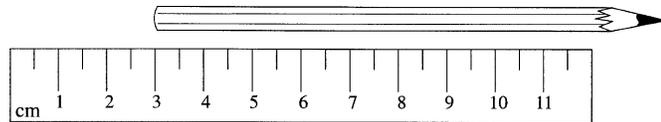
Which of these angles has a measure closest to 30° ?



Performance Category: Knowing

EXAMPLE ITEM 27
MEASUREMENT

Approximate length of pencil



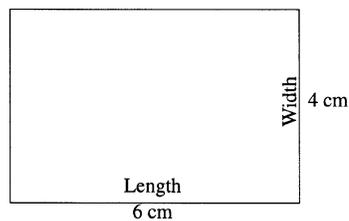
Which of these is closest to the length of the pencil in the figure?

- A. 9 cm
- B. 10.5 cm
- C. 12 cm
- D. 13.5 cm

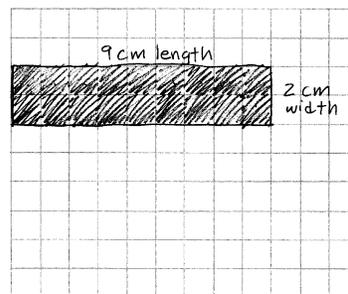
Performance Category: Using Complex Procedures

EXAMPLE ITEM 28
MEASUREMENT

New Rectangle



- a. In the space below, draw a new rectangle whose length is one and one half times the length of the rectangle above, and whose width is half the width of the rectangle above. Show the length and width of the new rectangle in centimeters on the figure.



- b. What is the ratio of the area of the new rectangle to the area of the first one?

Show your work.
 $3 \text{ to } 4$
 new $\Delta = 18 \text{ cm}^2 \div 3 = 6$ or $\frac{3}{4}$
 old $\Delta = 24 \text{ cm}^2 \div 3 = 8$

Performance Category: Solving Problems

WHAT HAVE STUDENTS LEARNED ABOUT PROPORTIONALITY?

A small set (11) of the mathematics items was designed to focus specifically on proportionality concepts and problems. Arguably, these items could have been classified in other content areas, usually fractions and number sense, but the decision was made to analyze them separately because they assess an important kind of mathematical reasoning. Example Items 29 through 33 illustrate these types of questions. The percent of correct responses for each country for the example items are provided in Table 3.6.

As described previously in Chapter 2, this item group was relatively more difficult for students than those for the other content areas. Figure 3.6 shows the extreme difficulty of these items for students. Only those students scoring above 600 on the mathematics scale were likely to answer most of these types of questions correctly.

Example Item 29, the least difficult of the items shown here, was one of the few proportionality items answered correctly by the majority of students in most countries. The item asked about adding 5 boys and 5 girls to a class that was three-fifths girls. On average, 62% of the students at seventh grade and 65% at eighth grade correctly answered that there would still be more girls than boys in the class.

Despite the overall difficulty encountered by students in this content area, there was an extremely large range in performance across countries. Example Item 32, requiring the students to determine the number of girls in a class of 28 based on the ratio of girls to boys, illustrates the extent of the difference in achievement levels. At the eighth grade, the question was answered correctly by 92% of the students in Singapore compared to very few in Colombia (12%), Greece (13%), South Africa (9%), and Kuwait (12%).

Table 3.6
**Percent Correct for Proportionality Example Items
Lower and Upper Grades (Seventh and Eighth Grades*)**

Country	Example 29 More boys or girls in class.		Example 30 Ratio of red paint in mixture.		Example 31 Amount paid for portion of items.	
	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade
[†] Belgium (Fl)	85 (2.1)	82 (2.9)	47 (2.4)	48 (2.4)	57 (3.4)	58 (4.1)
[†] Belgium (Fr)	74 (2.6)	76 (2.8)	45 (2.8)	49 (2.9)	34 (3.5)	41 (3.1)
Canada	68 (2.4)	66 (2.5)	46 (2.1)	56 (1.8)	22 (2.1)	26 (2.3)
Cyprus	59 (2.9)	63 (2.7)	35 (2.0)	34 (2.1)	21 (2.6)	30 (3.0)
Czech Republic	60 (3.5)	70 (2.7)	19 (1.9)	29 (1.9)	47 (3.3)	63 (2.8)
^{†2} England	66 (3.4)	69 (3.3)	34 (2.2)	39 (2.7)	14 (1.9)	17 (2.9)
France	66 (2.7)	75 (2.4)	48 (2.0)	51 (2.5)	38 (2.6)	54 (2.9)
Hong Kong	79 (2.1)	78 (1.7)	67 (2.8)	70 (2.4)	52 (3.3)	62 (3.2)
Hungary	60 (2.8)	67 (2.3)	29 (1.9)	36 (2.1)	30 (2.4)	42 (2.5)
Iceland	70 (3.4)	66 (4.6)	26 (2.2)	49 (4.1)	15 (2.7)	25 (4.1)
Iran, Islamic Rep.	51 (3.3)	51 (3.2)	27 (2.2)	31 (2.3)	15 (2.3)	19 (2.6)
Ireland	71 (2.7)	78 (2.4)	37 (1.9)	42 (2.3)	32 (2.8)	41 (3.3)
Japan	76 (1.9)	82 (1.9)	57 (1.5)	66 (1.4)	61 (2.2)	71 (2.0)
Korea	78 (2.1)	82 (2.2)	78 (1.8)	87 (1.4)	63 (2.3)	62 (2.5)
¹ Latvia (LSS)	44 (3.1)	57 (3.4)	23 (2.0)	27 (1.9)	25 (2.7)	39 (2.9)
¹ Lithuania	44 (3.1)	51 (3.0)	8 (1.2)	14 (1.5)	28 (3.4)	36 (3.2)
New Zealand	69 (2.5)	70 (2.3)	43 (2.3)	47 (1.9)	19 (2.4)	22 (2.0)
Norway	70 (4.2)	73 (2.4)	28 (2.2)	37 (2.0)	16 (2.5)	27 (2.4)
Portugal	39 (2.2)	50 (2.6)	16 (1.6)	21 (1.6)	9 (1.5)	20 (2.5)
Russian Federation	47 (3.1)	47 (2.5)	27 (2.0)	39 (2.6)	50 (2.5)	49 (3.8)
[†] Scotland	65 (2.4)	71 (2.7)	38 (2.2)	38 (2.2)	12 (2.0)	19 (2.6)
Singapore	83 (1.9)	85 (1.7)	89 (1.6)	95 (0.8)	79 (2.4)	83 (1.8)
Slovak Republic	57 (2.6)	62 (2.9)	24 (2.0)	32 (2.1)	38 (3.1)	54 (2.7)
Spain	63 (2.3)	62 (3.0)	24 (1.6)	34 (1.7)	30 (2.4)	42 (2.7)
Sweden	68 (2.5)	74 (2.0)	50 (2.1)	64 (1.7)	21 (2.2)	30 (2.0)
¹ Switzerland	73 (2.2)	76 (2.2)	39 (2.1)	42 (1.9)	47 (2.0)	60 (2.4)
[†] United States	58 (2.5)	62 (2.2)	45 (2.0)	53 (1.8)	18 (2.8)	23 (2.2)
Countries Not Satisfying Guidelines for Sample Participation Rates (See Appendix A for Details):						
Australia	71 (2.2)	74 (1.4)	41 (1.7)	42 (2.0)	21 (1.9)	31 (1.8)
Austria	69 (2.5)	73 (2.7)	21 (2.4)	21 (1.9)	56 (3.2)	67 (3.0)
Bulgaria	65 (5.4)	57 (4.4)	28 (3.2)	37 (3.8)	46 (8.5)	34 (4.4)
Netherlands	85 (2.7)	77 (2.7)	58 (2.8)	65 (2.7)	44 (4.7)	41 (3.7)
Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details):						
Colombia	26 (3.0)	30 (3.9)	14 (2.3)	15 (2.1)	3 (1.1)	7 (1.6)
^{††} Germany	70 (2.7)	67 (3.3)	26 (2.0)	26 (2.1)	29 (2.9)	37 (3.4)
Romania	48 (2.6)	52 (3.0)	29 (2.0)	39 (2.4)	30 (2.3)	32 (2.6)
Slovenia	62 (2.7)	66 (2.5)	29 (2.3)	39 (2.2)	39 (2.6)	52 (3.0)
Countries With Unapproved Sampling Procedures at Classroom Level (See Appendix A for Details):						
Denmark	54 (3.3)	68 (2.9)	30 (2.4)	31 (2.1)	16 (2.2)	28 (2.6)
Greece	55 (2.4)	59 (2.5)	41 (1.9)	50 (2.1)	33 (2.4)	39 (2.7)
[†] South Africa	32 (2.8)	31 (2.2)	18 (1.4)	16 (1.5)	2 (1.0)	2 (0.8)
Thailand	55 (2.4)	56 (2.7)	44 (2.2)	55 (2.4)	37 (2.9)	43 (2.9)
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):						
[†] Israel	–	75 (4.0)	–	39 (4.2)	–	42 (4.8)
Kuwait	–	25 (4.1)	–	14 (2.1)	–	2 (0.8)

*Seventh and eighth grades in most countries; see Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (–) indicates data are not available. Israel and Kuwait did not test at the seventh grade.

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1994-95.

Table 3.6 (Continued)**Percent Correct for Proportionality Example Items
Lower and Upper Grades (Seventh and Eighth Grades*)**

Country	Example 32 Number of girls from boy/girl ratio.		Example 33 Missing values in proportionality table.	
	Seventh Grade	Eighth Grade	Seventh Grade	Eighth Grade
[†] Belgium (Fl)	37 (2.6)	34 (3.7)	27 (2.5)	33 (2.9)
[†] Belgium (Fr)	38 (3.0)	48 (3.1)	14 (2.1)	19 (2.6)
Canada	28 (2.4)	43 (2.4)	24 (2.3)	26 (2.1)
Cyprus	18 (2.4)	24 (2.6)	18 (2.3)	24 (2.4)
Czech Republic	47 (3.3)	60 (3.7)	21 (3.1)	30 (3.2)
¹² England	40 (3.5)	42 (3.4)	15 (2.8)	18 (3.0)
France	29 (2.8)	43 (3.1)	30 (2.3)	33 (2.6)
Hong Kong	47 (3.3)	63 (3.3)	32 (2.3)	38 (2.9)
Hungary	37 (2.7)	57 (2.6)	19 (2.1)	24 (2.4)
Iceland	22 (3.3)	18 (3.1)	9 (2.0)	14 (3.2)
Iran, Islamic Rep.	19 (2.6)	22 (2.4)	20 (3.0)	31 (4.3)
Ireland	56 (2.9)	56 (2.9)	21 (2.1)	25 (2.1)
Japan	47 (1.9)	53 (1.8)	48 (2.2)	49 (2.2)
Korea	58 (3.1)	64 (2.6)	34 (3.1)	41 (2.6)
¹ Latvia (LSS)	21 (3.0)	32 (3.1)	12 (1.9)	21 (2.6)
¹ Lithuania	13 (2.7)	30 (2.7)	6 (1.4)	14 (2.2)
New Zealand	30 (2.7)	37 (2.5)	13 (1.8)	19 (2.1)
Norway	15 (2.2)	19 (2.2)	11 (1.8)	15 (1.8)
Portugal	8 (1.4)	17 (1.8)	19 (2.1)	21 (2.3)
Russian Federation	25 (2.1)	37 (3.1)	20 (2.5)	27 (2.3)
[†] Scotland	26 (2.6)	37 (3.3)	14 (2.2)	15 (2.4)
Singapore	89 (1.7)	92 (1.3)	42 (2.9)	47 (2.8)
Slovak Republic	46 (3.1)	58 (2.7)	27 (2.5)	27 (2.9)
Spain	14 (1.7)	24 (2.2)	16 (1.7)	10 (1.5)
Sweden	19 (2.0)	24 (2.0)	11 (1.4)	14 (1.8)
¹ Switzerland	26 (2.4)	38 (2.5)	20 (2.1)	29 (2.4)
[†] United States	27 (2.6)	34 (2.3)	19 (2.2)	20 (1.6)
Countries Not Satisfying Guidelines for Sample Participation Rates (See Appendix A for Details):				
Australia	33 (2.4)	50 (2.3)	18 (2.1)	22 (1.7)
Austria	42 (4.0)	46 (2.6)	15 (1.9)	18 (2.1)
Bulgaria	46 (5.5)	54 (4.3)	22 (4.9)	44 (6.4)
Netherlands	43 (3.5)	43 (4.6)	33 (3.3)	29 (3.1)
Countries Not Meeting Age/Grade Specifications (High Percentage of Older Students; See Appendix A for Details):				
Colombia	11 (3.4)	12 (2.0)	10 (1.9)	11 (2.2)
^{†1} Germany	19 (2.6)	30 (3.4)	11 (1.7)	18 (2.2)
Romania	22 (2.6)	29 (2.7)	22 (2.5)	29 (2.9)
Slovenia	19 (2.1)	43 (2.7)	17 (2.5)	24 (2.1)
Countries With Unapproved Sampling Procedures at Classroom Level (See Appendix A for Details):				
Denmark	25 (3.1)	35 (3.5)	10 (1.9)	13 (2.3)
Greece	10 (1.5)	13 (1.9)	26 (2.6)	30 (2.3)
[†] South Africa	5 (1.5)	9 (1.7)	13 (1.3)	13 (1.4)
Thailand	37 (2.7)	48 (2.7)	36 (2.3)	39 (2.5)
Unapproved Sampling Procedures at Classroom Level and Not Meeting Other Guidelines (See Appendix A for Details):				
¹ Israel	–	22 (3.4)	–	17 (2.8)
Kuwait	–	12 (3.5)	–	15 (2.0)

*Seventh and eighth grades in most countries; see Table 2 for information about the grades tested in each country.

[†]Met guidelines for sample participation rates only after replacement schools were included (see Appendix A for details).

¹National Desired Population does not cover all of International Desired Population (see Table A.2). Because coverage falls below 65%, Latvia is annotated LSS for Latvian Speaking Schools only.

²National Defined Population covers less than 90 percent of National Desired Population (see Table A.2).

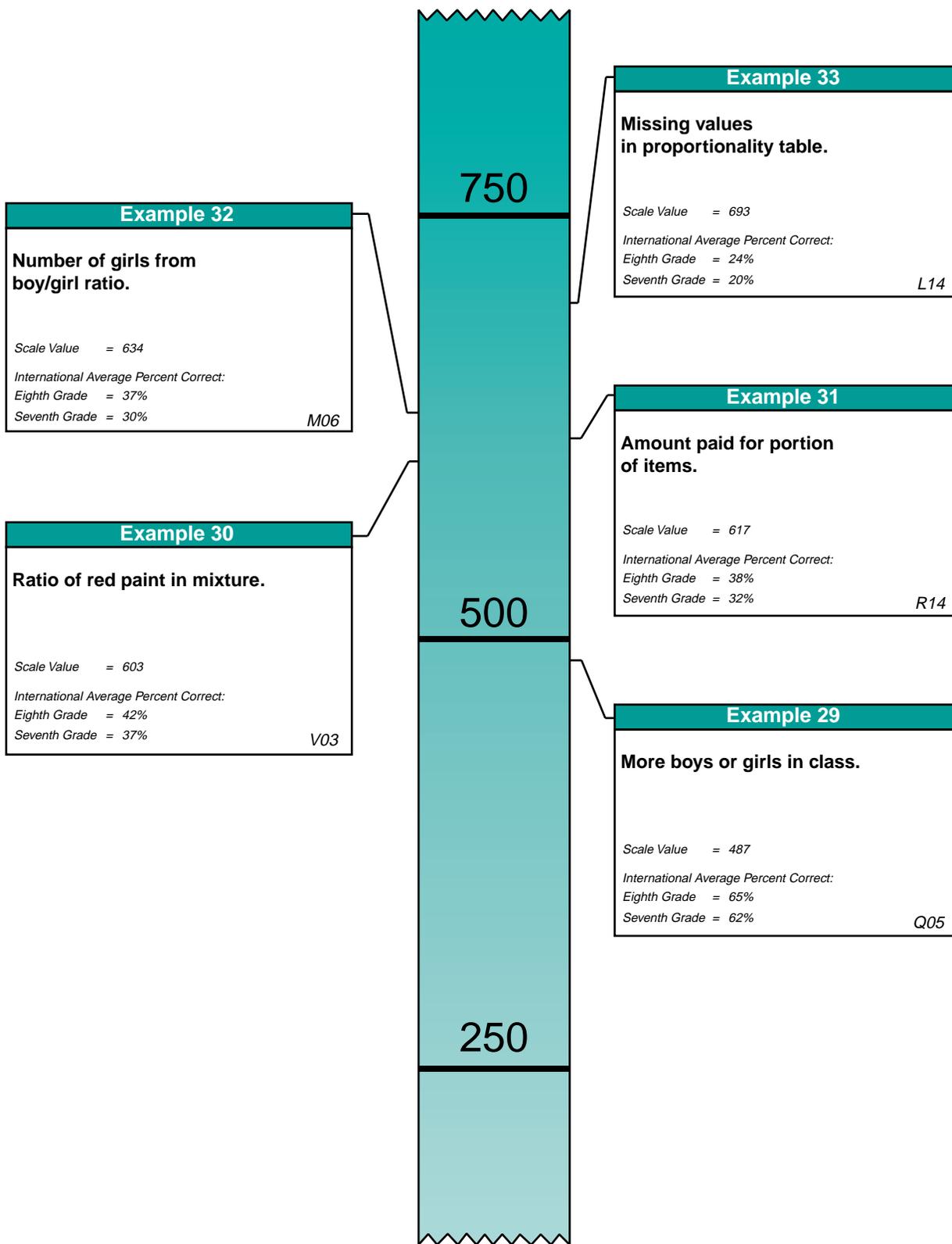
() Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

A dash (–) indicates data are not available. Israel and Kuwait did not test at the seventh grade.

SOURCE: IEA Third International Mathematics and Science Study (TIMSS), 1994-95.

Figure 3.6

**International Difficulty Map for Proportionality Example Items
Lower and Upper Grades (Seventh and Eighth Grades*)**



*Seventh and eighth grades in most countries; see Table 2 for information about the grades tested in each country.
NOTE: Each item was placed onto the TIMSS international mathematics scale based on students' performance in both grades. Items are shown at the point on the scale where students with that level of proficiency had a 65 percent probability of providing a correct response.

EXAMPLE ITEM 29
PROPORTIONALITY**More boys or girls in class**

Three-fifths of the students in a class are girls. If 5 girls and 5 boys are added to the class, which statement is true of the class?

- A. There are more girls than boys.
- B. There are the same number of girls as there are boys.
- C. There are more boys than girls.
- D. You cannot tell whether there are more girls or boys from the information given.

Performance Category: Solving Problems

EXAMPLE ITEM 30
PROPORTIONALITY**Ratio of red paint in mixture**

To mix a certain color of paint, Alana combines 5 liters of red paint, 2 liters of blue paint, and 2 liters of yellow paint. What is the ratio of red paint to the total amount of paint?

- A. $\frac{5}{2}$
- B. $\frac{9}{4}$
- C. $\frac{5}{4}$
- D. $\frac{5}{9}$

Performance Category: Performing Routine Procedures

EXAMPLE ITEM 31 PROPORTIONALITY

Amount paid for portion of items

Peter bought 70 items and Sue bought 90 items. Each item cost the same and the items cost \$800 altogether. How much did Sue pay?

Answer: Sue paid \$450

$$\begin{array}{r} 16 \overline{)800} \\ 3 \\ \underline{48} \\ 320 \\ \underline{300} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

Performance Category: Solving Problems

EXAMPLE ITEM 32 PROPORTIONALITY

Number of girls from boy/girl ratio

A class has 28 students. The ratio of girls to boys is 4 : 3. How many girls are in the class?

Answer: 16

$$\frac{28}{7} \times 4 = 4 \times 4$$

Performance Category: Solving Problems

EXAMPLE ITEM 33 PROPORTIONALITY

Missing values in proportionality table

The table shows the values of x and y , where x is proportional to y .

x	3	6	P
y	7	Q	35

What are the values of P and Q ?

- A. $P = 14$ and $Q = 31$
- B. $P = 10$ and $Q = 14$
- C. $P = 10$ and $Q = 31$
- D. $P = 14$ and $Q = 15$
- E. $P = 15$ and $Q = 14$

Performance Category: Performing Routine Procedures